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AN EQUIVARIANT VERSION OF GROMOV'S THEOREM

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In this note we announce an equivariant version of the theorem of Gromov [2], [3], [7] concerning the classification of smooth sections of a differentiable fibre bundle whose r-jets satisfy an "intrinsic differential inequality". The development of Gromov's theorem began with the Smale-Hirsch theory of immersions [8], [5], which was clarified and generalized by Phillips [6], Haefliger and Poenaru [4] and Gromov. Phillips' submersion theorem makes clear the essential role played by the assumption that the source manifold is nonclosed (i.e., no compact component meets the boundary); in fact the immersion theorem in positive codimension can be deduced from the submersion theorem using the (nonclosed) normal bundle of the source manifold in the target.

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Preliminaries on G-fibre bundles. Throughout this paper G denotes a compact Lie group. A *G-manifold* is a differentiable (C^∞) manifold X together with a differentiable action of G on X. Let X be a G-manifold and $p:E \to X$ a (locally trivial) differentiable fibre bundle. If there is a differentiable action of G on E such that each $g \in G$ operates as a bundle map over the given map $g:X \to X$, then we say that $p:E \to X$ is a (differentiable) *G-fibre bundle* (when p has a specified Lie structure group, bundle maps are understood to be induced by principal bundle maps). For example, the projection $p:X \times Y \to X$ from a product of G-manifolds with the diagonal action is equivariant, and G acts as a group of bundle maps if we consider p a trivial fibre bundle with structure group G.

A differentiable G-fibre bundle $p:E \to X$ is called *G-locally trivial* if for each $x \in X$ there is a G_x-invariant neighbourhood U_x of x (G_x is the isotropy subgroup of x) such that $p \mid U_x$ is differentiably G_x-equivariantly equivalent to the trivial G_x-fibre bundle $U_x \times p^{-1}(x)$. G-local triviality allows us to work equivariantly in local coordinates. Though differentiable

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