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HOLOMORPHIC LEFSCHETZ FIXED POINT FORMULA

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1. Let X be an n-dimensional complex analytic manifold and $\varphi : X \to X$ a holomorphic map. Let Ω be the sheaf of germs of holomorphic functions on X and $H^i(X, \Omega)$ the ith cohomology group of X with coefficients in the sheaf Ω. The map φ defines endomorphisms, $H^i(\varphi)$ of $H^i(X, \Omega)$, $i \geqq 0$. Let $L(\varphi)$ be the Lefschetz number defined by

$$L(\varphi) = \sum_{i=0}^{n} (-1)^i \operatorname{trace} H^i(\varphi).$$

We are concerned with the problem of computing $L(\varphi)$.

REMARK. Let G be a compact Lie group acting on X as a group of holomorphic diffeomorphisms and $\varphi \in G$. The problem in this case has been solved by Atiyah and Singer, see [**2**]. Also in the case φ has isolated fixed points, the problem was solved in the nondegenerate case (see §2 for definition) by Atiyah and Bott in [**1**] and by Toledo and Tong in [**6**] and [**7**] in the degenerate case.

2. **The statement of main theorem.** Let X_φ be the fixed point set of the map φ, $X_\varphi = \{x \in X \text{ s.t. } \varphi(x) = x\}$. We start by stating the conditions under which we have been able to compute the Lefschetz number $L(\varphi)$.

(C_1) X_φ is a complex analytic submanifold of X and moreover with this complex analytic structure, X_φ is a Kähler manifold.

Let us write X_φ as a finite union of closed connected submanifolds of X:

$$X_\varphi = \bigcup_{i=1}^{N} Y_i. \tag{1}$$

Let $\lambda_1^i, \ldots, \lambda_{m_i}^i$ be the eigenvalues of the endomorphism $(\varphi_*)_z$ of $T_z(X)$, $z \in Y_i$, with multiplicities $n_1^i, \ldots, n_{m_i}^i$; eigenvalues λ_j^i are independent of $z \in Y_i$ because of the holomorphic nature of the situation. If 1 is an eigenvalue of the map φ_* we take $\lambda_1^i = 1$.

The vector bundles $T(X)|_{Y_i}$ decompose as a direct sum of holomorphic vector subbundles E_j^i $(1 \leqq j \leqq m_i)$ whose fibres $(E_j^i)_z$ are defined by:

$$(E_j^i)_z = \{v \in T_z(X) \text{ s.t. } (\varphi_* - \lambda_j^i I)^{n_j^i} v = 0\}.$$

We now state our other conditions.

(C_2) The fixed points are nondegenerate: 1 is an eigenvalue of

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