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INCIDENCE ALGEBRAS AS ALGEBRAS OF ENDOMORPHISMS

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1. **Introduction.** The order filters on a locally finite partially ordered set P constitute the open sets for a topology on P. A sheaf of abelian topological groups will be constructed on the topological space P, and the endomorphism ring of this sheaf will be proved isomorphic to the incidence algebra of P (over $\mathbf{Z}$).

2. **A sheaf of abelian groups on** P. Let P be a locally finite (every interval $[x, y]$ of P is finite) partially ordered set. An (order) filter on P is a subset V of P which contains y whenever $x \leqq y$ and $x \in V$. For $x \in P$,

$$V_x = \{y \in P : x \leqq y\}$$

is the principal filter generated by x. The filters on P are easily seen to be the open sets for a topology on P, and the increasing maps from P to another locally finite partially ordered set Q are precisely the continuous functions from P to Q [**5**].

For each filter V on P, let $M(P, V)$, or simply $M(V)$ when reference to P is understood, denote the free abelian group on V. For filters $U \subseteq V$, let $r(V, U): M(V) \to M(U)$ be the group homomorphism determined by

$$\begin{aligned} x &\mapsto x \quad \text{if } x \in U, \\ x &\mapsto 0 \quad \text{if } x \notin U, \end{aligned}$$

for $x \in V$.

PROPOSITION 1. *M (with the restriction maps $r(V, U)$) is a sheaf of abelian groups on P.*

PROOF. M is easily seen to be a presheaf of abelian groups. For any open cover $V = \bigcup V_i$, consider

$$M(V) \xrightarrow{\pi} \prod M(V_i) \underset{\pi_2}{\overset{\pi_1}{\rightrightarrows}} \prod M(V_k \cap V_j)$$

where π is induced by the restrictions $r(V, V_i)$, π_1 is induced by the restrictions $r(V_k, V_k \cap V_j)$, and π_2 is induced by the restrictions $r(V_j, V_k \cap V_j)$. That π is injective is clear. Let $\alpha = (\alpha_i) \in \prod M(V_i)$ where $\alpha_i = \sum \alpha_{i,x} x$, and suppose that $\pi_1(\alpha) = \pi_2(\alpha)$. Then $\alpha_{k,x} = \alpha_{j,x}$ for any $x \in V_k \cap V_j$. So

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