

MEASURES WHOSE TRANSFORMS VANISH AT INFINITY

BY LOUIS PIGNO AND SADAHIRO SAEKI

Communicated by Bertram Yood, January 8, 1973

Let G be an LCA group with dual Γ , $M(G)$ the usual convolution algebra of finite Borel measures on G and $\hat{\cdot}$ the Fourier-Stieltjes transformation. By $M_0(G)$ we mean the ideal of measures $\mu \in M(G)$ such that $\hat{\mu}$ vanishes at ∞ . The purpose of this note is to announce the following results.

THEOREM. *Let G be a nondiscrete LCA group with Haar measure m_G . Let λ be a nonzero measure in $M_0^+(G)$ and D a σ -compact subset of G with $m_G(D) = 0$. Then there exists a nonzero measure σ in $M_0^+(\text{supp } \lambda)$ such that*

- (i) $\text{supp } \sigma$ is compact and has λ -measure zero,
- (ii) $m_G[D + G_p(\text{supp } \sigma)] = 0$.

Here $G_p(\text{supp } \sigma)$ denotes the subgroup of G which is algebraically generated by $\text{supp } \sigma$.

COROLLARY 1 (VAROPOULOS [2]). *Every nondiscrete G contains a compact perfect set E such that $M_0^+(E) \neq \{0\}$ and $m_G[G_p(E)] = 0$.*

Let $M_s(G)$ denote the set of measures singular with respect to Haar measure on G .

COROLLARY 2. *Let B be a separable subset of $M_s(G) \setminus \{0\}$ such that $\hat{\mu}\hat{F} \neq 0$ for all $\mu \in B$ and some σ -compact subset \hat{F} of Γ . Then there exists a measure $\sigma \in M_s^+(G)$ such that*

$$\bigcup_{n=1}^{\infty} (B * \sigma^n) \subset M_s(G) \setminus \{0\}.$$

COROLLARY 3. *Suppose $\mu \in M(G)$ has the property that, $\forall \sigma \in M_0^+(G)$, $\exists n = n_\sigma \in N$ (the natural numbers) such that $\mu * \sigma^n \in L^1(G)$. Then $\mu \in L^1(G)$. In particular, we have $\mu * M_0(G) \subset L^1(G) \Rightarrow \mu \in L^1(G)$.*

As an application of Corollary 3 we give the solution to a question implicit in Meyer [1, p. 94]. Let E be a subset of Γ and define

$$M_{\hat{E}}(G) = \{\mu \in M(G) : \text{supp } \hat{\mu} \subset E\}.$$

We say that \hat{E} is a Riesz set of type 0 if

$$(a) \quad M_0(G) \wedge |_{\hat{E}} = L^1(G) \wedge |_{\hat{E}}.$$

Note that every Sidon set (or Helson set) has property (a).