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AN EXTENSION OF A THEOREM OF HAMADA ON THE CAUCHY PROBLEM WITH SINGULAR DATA[1]

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Introduction. Hamada [1] proved the following result about the propagation of singularities in the Cauchy problem for an analytic linear partial differential operator. Assume that the initial data are analytic at the point $\mathbf{0}$ except for singularities along a submanifold T of the initial surface containing $\mathbf{0}$. Let $K^{(1)}, \cdots, K^{(m)}$ be the characteristic surfaces of the operator emanating from T. Under the assumption that the $K^{(i)}$ have multiplicity one, he showed that the solution of the Cauchy problem is analytic at $\mathbf{0}$ except for logarithmic singularities along the $K^{(i)}$. We extend his result to the case where the $K^{(i)}$ have constant multiplicity.

1. **Definitions and theorem.** Let $\mathbf{C}^{n+1}$ denote the set of $(n+1)$-tuples $\mathbf{x} = (x^0, \ldots, x^n)$ of complex numbers. Let S be an n-dimensional analytic submanifold of $\mathbf{C}^{n+1}$, and let T be an $(n-1)$-dimensional analytic submanifold of S. Since our results are local, we can assume $S = \{(0, x^1, \ldots, x^n) \in \mathbf{C}^{n+1}\}$ and $T = \{(0, 0, x^2, \ldots, x^n) \in \mathbf{C}^{n+1}\}$.

Let $D_i = \partial/\partial x^i, \mathbf{D} = (D_0, \ldots, D_n)$, and let $a : \mathbf{x} \to a(\mathbf{x}; \mathbf{D})$ be an analytic partial differential operator on a neighborhood of $\mathbf{0}$ in $\mathbf{C}^{n+1}$. Let $h(\mathbf{x}; \mathbf{D})$ be the principal part of $a(\mathbf{x}; \mathbf{D})$. We assume that S is not a characteristic surface of a at $\mathbf{0}$, so $h(\mathbf{0}; 1, 0, \ldots, 0) \neq 0$. Let $\mathbf{p} = (p_0, \ldots, p_n)$ be an $(n+1)$-tuple of formal variables, so $h(\mathbf{x}; \mathbf{p})$ is a homogeneous polynomial in $\mathbf{p}$ with analytic coefficients.

We say that the operator a has *constant multiplicity* at $\mathbf{0}$ in the direction of T if we can factor h as

$$h(\mathbf{x}; \mathbf{p}) = [h_1(\mathbf{x}; \mathbf{p})]^{k_1} \cdots [h_s(\mathbf{x}; \mathbf{p})]^{k_s}$$

for all $\mathbf{x}$ in a neighborhood of $\mathbf{0}$, where each $h_i(\mathbf{x}; \mathbf{p})$ is a polynomial in $\mathbf{p}$ of degree m_i with analytic coefficients, and the Σm_i roots of the polynomials $h_i(\mathbf{0}; \tau, 1, 0, \ldots, 0)$ in τ are all distinct. If $s = k_1 = 1$, then a is said to be of *multiplicity one* at $\mathbf{0}$ in the direction of T.

Assume now that a has constant multiplicity at $\mathbf{0}$ in the direction of T. It can be shown that we can find $m = \Sigma m_i$ analytic *characteristic functions* $\varphi^{(1)}, \ldots, \varphi^{(m)}$ of h defined in a neighborhood N of $\mathbf{0}$ satisfying:

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[1]The results described here are contained in the author's 1972 Ph.D. dissertation, written at Brandeis University under the supervision of Professor Richard Palais.