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# COHOMOLOGY OF BRAID SPACES

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1. **Introduction and results.** Let $M$ be a manifold; define $F(M, k)$ to be the subspace $\{\langle x_1, \ldots, x_k\rangle | x_i \in M,\ x_i \neq x_j \text{ if } i \neq j\}$ of $M^k$. There is a proper right action of $\Sigma_k$, the symmetric group on $k$-letters, on $F(M, k)$ given by

$$\sigma \cdot \langle x_1, \ldots, x_k\rangle = \langle x_{\sigma(1)}, \ldots, x_{\sigma(k)}\rangle, \qquad \sigma \in \Sigma_k.$$

Let $B(M, k)$ denote the orbit space $F(M, k)/\Sigma_k$. The object of this paper is to outline the calculation of

$$H^*(\mathrm{Hom}_{\Sigma_p}(C_*F(\mathbf{R}^n, p); \mathbf{Z}_p(q))), \qquad n \geqq 2, p \text{ prime},$$

where $C_*F(\mathbf{R}^n, p)$ denotes the singular chains of $F(\mathbf{R}^n, p)$, and $\mathbf{Z}_p(q)$ denotes the $\Sigma_p$-module $\mathbf{Z}_p$ with $\Sigma_p$-action $\sigma \cdot x = (-1)^{qs(\sigma)}x$ for $x \in \mathbf{Z}_p$ and $\sigma \in \Sigma_p$ ($(-1)^{s(\sigma)}$ is the sign of $\sigma$). Since the $\Sigma_p$-action on $F(\mathbf{R}^n, p)$ is proper, we may identify $H^*(\mathrm{Hom}_{\Sigma_p}(C_*F(\mathbf{R}^n, p); \mathbf{Z}_p(2q)))$ with $H^*(B(\mathbf{R}^n, p); \mathbf{Z}_p)$ [**5**]. By abuse of notation we denote $H^*(\mathrm{Hom}_{\Sigma_p}(C_*F(\mathbf{R}^n, p); \mathbf{Z}_p(q)))$ by $H^*(B(\mathbf{R}^n, p); \mathbf{Z}_p(q))$.

The interest in $H^*(B(\mathbf{R}^n, p); \mathbf{Z}_p(q))$ arises from the work of Peter May [**6**], [**7**] which implies that each class in $H_*(B(\mathbf{R}^n, p); \mathbf{Z}_p(q))$ determines a homology operation on all classes of degree $q$ in the mod $p$ homology of any $n$-fold loop space.

For our calculations, we rely heavily on the map of fibrations

$$\begin{array}{ccc} \Sigma_p & \longrightarrow & \Sigma_p \\ \downarrow & & \downarrow \\ F(\mathbf{R}^n, p) & \xrightarrow{\hat{f}} & F(\mathbf{R}^\infty, p) \\ \downarrow & & \downarrow \\ B(\mathbf{R}^n, p) & \xrightarrow{f} & B(\mathbf{R}^\infty, p) \end{array}$$

where $F(\mathbf{R}^\infty, p) = \varinjlim F(\mathbf{R}^n, p)$ and $B(\mathbf{R}^\infty, p) = F(\mathbf{R}^\infty, p)/\Sigma_p$. Here $f$ and $\hat{f}$ are the evident inclusions. Since $F(\mathbf{R}^\infty, p)$ is contractible with free $\Sigma_p$-action, $B(\mathbf{R}^\infty, p)$ is a $K(\Sigma_p, 1)$. Obviously

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