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AN ALGEBRAIC PROOF OF AN ANALYTIC RESULT OF SHUCK'S

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In [1] the authors pose the following problem: Let $F(X_1, \ldots, X_r)$ be a polynomial in r variables with integer coefficients. Let p be a prime. For each positive integer n let c_n be the number of solutions modulo p^n of the congruence $F(x_1, \ldots, x_r) \equiv 0 \bmod (p^n)$. Form the Poincaré series of F

$$P_F(t) = \sum_{n=1}^{\infty} c_n t^n.$$

Is $P_F(t)$ a rational function in t?

John Shuck [3] generalized this problem and then answered it affirmatively in the "nonsingular" case. His approach was to develop a calculus on manifolds over p-adic fields (including a theory of integration on submanifolds, Fubini's theorem, and change of variables), and then to utilize analytic techniques for counting.

In the following we present another reformulation of the problem, this time in an algebraic geometric setting, and give a purely algebraic and quite elementary proof in the smooth case.

My thanks here are to M. J. Greenberg who insisted that this should be written down.

1. **A reformulation.** We consider the following situation: S is a scheme (that is, a prescheme in the sense of EGA), X is an S-scheme, I a quasi-coherent ideal of O_S. Let S_n be the subscheme of S defined by the ideal I^n. Then we have the cartesian diagram

$$\begin{array}{ccc} X & \xleftarrow{\;j_n\;} & X_n \\ {\scriptstyle f}\downarrow & & \downarrow{\scriptstyle f_n} \\ S & \xleftarrow[\;i_n\;]{} & S_n \end{array}$$

Let $\Gamma_n = \operatorname{Hom}_{S_n}(S_n, X_n) = \operatorname{Hom}_S(S_n, X)$. We pose the following problem: If $c_n = \operatorname{card}(\Gamma_n)$ is finite for all n greater than some N, set

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