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INDUCTION AND STRUCTURE THEOREMS FOR GROTHENDIECK AND WITT RINGS OF ORTHOGONAL REPRESENTATIONS OF FINITE GROUPS

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ABSTRACT. The Grothendieck- and Witt- ring of orthogonal representations of a finite group is defined and studied. The main application (only indicated) is the reduction of the computation of Wall's various L-groups for a finite group π to those subgroups of π, which are a semidirect product of a cyclic group γ of odd order with a 2-group β, such that any element in β acts on γ either by the identity or by taking any element in to its inverse.

Let π be a finite group and R a Dedekind ring. An $R\pi$-lattice (M, f) or just M is defined to be a finitely generated, R-projective $R\pi$-module M together with a symmetric, π-invariant nonsingular form $f: M \times M \to R$ (cf. [**3**]). For two $R\pi$-lattices M_1 and M_2 one has their orthogonal sum $M_1 \perp M_2$ and tensor product $M_1 \otimes M_2$, thus the isomorphism classes of $R\pi$-lattices form a half-ring $Y^+(R, \pi)$, whose associated Grothendieck ring is denoted by $Y(R, \pi)$. For a subgroup $\gamma \leqq \pi$ one has in an obvious way, restriction and induction of $R\pi$-lattices, resp. $R\gamma$-lattices, and it is easily seen (cf. [3]) that this makes $Y(R,\text{-})$ into a G-functor in the sense of Green (cf. [**5**]).

As in the theory of integral group-representations, where the Grothendieck ring of isomorphism classes of $R\pi$-modules is much too large for many purposes and is thus replaced by its quotient $G_0(R, \pi)$ (in the sense of [**9**]) modulo the ideal, generated by the Euler characteristics of short exact sequences of $R\pi$-modules, we are going to define certain quotients of $Y(R, \pi)$, using a relation which was first introduced by D. Quillen in [7, §5]. At first let us remark, that for any finitely generated R-projective $R\pi$-module N, one has the associated hyperbolic module $H(N) = (N \oplus N^*, f)$ with $N^* = \mathrm{Hom}_R(N, R)$ the R-dual of N, considered as $R\pi$-module (with $(g \cdot v)(n) = v(g^{-1} \cdot n)$, $g \in \pi$, $v \in N^*$, $n \in N$) and $f(N, N) = f(N^*, N^*) = 0$, $f(n, v) = v(n)$, $n \in N$, $v \in N^*$. We now define a Quillen pair (M, N) to be an $R\pi$-lattice $M = (M, f)$ together with an

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