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UNIQUENESS AND INTERPOLATION RESULTS FOR ENTIRE FUNCTIONS OF EXPONENTIAL TYPE[1]

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1. **Introduction.** Let $K[\Omega]$ denote the collection of entire functions of exponential type whose Borel transforms are analytic on Ω^c, the complement of Ω taken relative to the sphere. The domain Ω is assumed to be simply connected throughout the article. This note discusses a new approach for proving theorems about uniqueness classes of the form $K[\Omega]$ for sequences of linear functionals $\{L_n\}$, where

$$L_n(f) = \frac{1}{2\pi i}\int_\Gamma g_n(\lambda)F(\lambda)\,d\lambda \qquad (n = 0, 1, \ldots). \tag{1.1}$$

Each g_n $(n = 0, 1, \ldots)$ is in $H(\Omega)$, the collection of all functions holomorphic on Ω, F denotes the Borel transform of f, and Γ is a simple closed contour in Ω with F holomorphic outside and on Γ. We shall also characterize sequences of complex numbers $\{b_n\}$ which can be interpolated by $K[\Omega]$ relative to $\{L_n\}$.

Our results on uniqueness and interpolation require the following setting. Place the topology of uniform convergence on compact subsets of Ω on $H(\Omega)$. Then the dual space of $H(\Omega)$ can be identified with the collection of functions holomorphic on Ω^c which vanish at ∞ [**8**]. This collection will be denoted by $H_0(\Omega^c)$. Note that $H_0(\Omega^c)$ is also the collection of Borel transforms of all functions in $K[\Omega]$. The bilinear form relating $H(\Omega)$ and $H_0(\Omega^c)$ is given by

$$\langle \alpha, F\rangle = \frac{1}{2\pi i}\int_\Gamma \alpha(\lambda)F(\lambda)\,d\lambda$$

for each α in $H(\Omega)$ and each F in $H_0(\Omega^c)$.

Let Ω_λ and Ω_ζ be simply connected domains in the plane and let T denote a continuous linear operator from $H(\Omega_\lambda)$ to $H(\Omega_\zeta)$. For $\{g_n\}$ with each g_n in $H(\Omega_\lambda)$ define $\{L_{T_n}\}$ by

$$L_{T_n}(f) = \frac{1}{2\pi i}\int_\Gamma [T(g_n)](\zeta)F(\zeta)\,d\zeta \qquad (n = 0, 1, \ldots)$$

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[1] These results are contained in the author's 1972 Ph.D. dissertation at the University of Cincinnati, written under the supervision of Professor Richard F. DeMar.