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MODULES OVER DIFFERENTIAL POLYNOMIAL RINGS

BY RICHARD E. BLOCK[1]

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This note announces a number of results on the structure of differential modules over differential rings, where differential ring means a ring with a family of derivations and differential module means a module having a family of operators compatible with the derivations of the ring. To fix notation, throughout the paper we let A denote an associative ring, $M = {}_AM$ an A-module, λ the corresponding representation of A in M, and $D = \{d_i\}_{i \in I}$ a family of derivations of A (to A). We are concerned with the structure of M when M is D-admissible where we call M D-admissible if the representation λ can be extended to a representation ρ of $A[Y; D]$ in M. Here $A[Y; D]$ denotes the ring of differential polynomials in the (noncommuting) indeterminates $Y = \{y_i\}_{i \in I}$; thus $A[Y; D]$ is the free (left) module over A with basis the free monoid on Y, with multiplication determined by the relations $\{y_ia - ay_i = d_ia | a \in A, i \in I\}$. For example, it can be shown that every projective A-module is D-admissible.

The main theorems announced here concern those D-admissible M for which the representation ρ of $A[Y; D]$ in M can be chosen to be irreducible; such M will be completely determined under any of several finiteness conditions on A or on M. The results are new even in the finite-dimensional case. While the results are about associative rings, they are in part motivated by the theory of Lie algebras. In fact, the main theorems can be regarded as generalizations of some classical results on Lie algebras. While these classical results fail hopelessly at prime characteristic, we shall see that associative versions of them do hold in a sharp form at characteristic p.

Differentiably irreducible (d.i.) modules. Recall that an additive mapping t of M to M is called a *differential transformation* of M if there is a mapping d of A to A such that $ta - at = \lambda(da)$ for all a in A; we then say that t is a differential transformation of M with respect to d. If M is faithful then d is necessarily a derivation of A. If for each i in the index set I there is a differential transformation t_i with respect to the given derivation d_i, then M is D-admissible, and conversely (take $\rho(y_i) = t_i$).

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