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EXTENDING EQUIVARIANT MAPS FOR COMPACT LIE GROUP ACTIONS[1]

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ABSTRACT. Extensions of maps are studied in the category of spaces with actions of a compact Lie group G. If G acts on a finite dimensional compact metric space X with a finite number of conjugacy classes of isotropy subgroups, if $\tilde{X}$ is a closed equivariant subspace of X such that the action on $X - \tilde{X}$ is free and if $f:\tilde{X} \to Y$ is an equivariant map to a compact metric space Y with a G-action, then an equivariant neighborhood extension of f exists provided that Y is an ANR; if Y is an AR, then f can be equivariantly extended over X.

1. **Introduction.** In previous papers [**2**] and [**3**], an extension theorem for equivariant maps in the category of spaces with periodic homeomorphisms was proved. That theorem was then applied to a characterization of equivariant absolute neighborhood retracts and absolute retracts in this category. The purpose of this note is to announce results which extend some of the results of [**2**] and [**3**] from the category of $\boldsymbol{Z}_p$-actions to the category of compact Lie group actions. Detailed proofs will appear in a forthcoming paper.

The following theorem is the main result of this paper.

(1.1) THEOREM. *Let G be a compact Lie group acting on a finite dimensional compact metric space X with a finite number of conjugacy classes of isotropy subgroups; and let $\tilde{X}$ be a closed equivariant subspace of X containing all the fixed points of the elements of G different from the identity. Let G act on a compact metric space Y and let $f:\tilde{X} \to Y$ be an equivariant map. Then*:

(i) *If Y is an ANR, there exists an equivariant extension $g:U \to Y$ of f over an equivariant neighborhood U of $\tilde{X}$ in X*;

(ii) *If Y is an AR, there exists an equivariant extension $g:X \to Y$ of f over X.*

As it was pointed out in [**2**] and [**3**], the problem of equivariant extension maps is not trivial even for $\boldsymbol{Z}_2$-actions, that is, for spaces with involutions, if they are not fixed point free. The significance of this result

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[1] This work was done while the author held a visiting professorship at the University of Heidelberg in the summer term of 1972.