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# FORMAL $A$-MODULES[1]

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1. **Introduction.** In this announcement, we present results obtained in an investigation of *one-parameter formal A-modules* defined over a $p$-adic integer ring $B$. The formal modules were introduced into the literature by Lubin and Tate in [6] wherein the formal modules of height one and their endomorphisms were used to describe the maximal Abelian extension and the Artin symbol of an arbitrary local field. Since then, they have gone virtually unnoticed, except for [5]. Aside from adding enrichment to the theory of formal groups in general, it is believed that a study of the formal modules will bear fruit in the form of applications to number theory, in the spirit of [6]. I wish to acknowledge my gratitude to Professor Jonathan Lubin for his insights and suggestions in the development and presentation of this work.

2. **Formal groups and modules.** Throughout this paper, $Z_p$ will denote the ring of $p$-adic integers; $A$ and $B$ will be fixed (integrally closed) complete $p$-adic integer rings with $Z_p \subseteq A \subseteq B$. Let $k$ be the residue class field of $A$ and assume that $k$ contains $q$ elements. A (one-parameter) *formal group law* $F(X, Y)$ defined over $B$ is a formal power series $F(X, Y) \in B[[X, Y]]$ for which (i) $F(X, 0) = X$, and (ii) $F(F(X, Y), Z) = F(X, F(Y, Z))$. As $B$ contains no nilpotent elements, it results that (iii) $F(X, Y) = F(Y, X)$. If $G(X, Y)$ is another formal group law defined over $B$, then a *$B$-homomorphism $t(x)$ from $F$ to $G$* is a power series $t(x) \in B[[X]]$ without constant term such that $t(F(X, Y)) = G(t(X), t(Y))$. If $t(x)$ is invertible as a power series, we say $t(x)$ is a *$B$-isomorphism*; and if $t(x) \equiv x \bmod \deg 2$, we say $t(x)$ is a *strong $B$-isomorphism.* It results (cf. [4]) that under $F$-addition and composition of power series, End($F$), the set of all $B$-endomorphisms of $F(X, Y)$, becomes a complete topological ring for which $Z_p \subseteq \mathrm{End}(F) \subseteq B$.

DEFINITION. A one-parameter formal group law $F(X, Y)$ defined over $B$ is a (one-parameter) *formal $A$-module* if for each $a \in A$ there is a $B$-endomorphism $[a]_F(x)$ of $F(X, Y)$ with $[a]_F(X) \equiv aX \bmod \deg 2$.

Let $L$ and $K$ be the fields of fractions of $A$ and $B$, respectively, let $B^*$ denote the multiplicative group of units of $B$, and let $\pi$ be a fixed prime element of $A$. It results that $[p]_F(x)^*$, the reduction of $[p]_F(x)$ to the residue class field of $B$, is either equal to the zero power series or else is a power series in $x^{p^H}$ whose first nonzero coefficient occurs in degree $p^H$, for

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