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HOMOTOPY EQUIVALENCES WHICH ARE CELLULAR AT THE PRIME 2

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0. We consider complexes having the property that the link of each point has the homology of S^{n-1} with coefficients in $Z[1/\mathrm{odd}]$. Such complexes are called homology n-manifolds at the prime 2. Henceforth, all such spaces will be assumed to be 4-connected.

We state some salient facts.

LEMMA 1[**a**]. *Let K^n be a 4-connected Poincaré duality space, v its Spivak normal fibration and $T(v)$ the corresponding Thom spectrum. There is a spectrum $\mathscr{W}(v)$ and a map $p:\mathscr{W}(v) \to T(v)$ such that v is fiberhomotopically equivalent to a PL bundle if and only if p admits a section $s: T(v) \to \mathscr{W}(v)$.*

We remind the reader of the construction of $\mathscr{W}(v)$ in §1 below, where we construct another spectrum $\mathscr{W}_{(2)}(v)$, with an obvious natural map $f:\mathscr{W}(v) \to \mathscr{W}_{(2)}(v)$ such that p factors as $\mathscr{W}(v) \to^{f} \mathscr{W}_{(2)}(v) \to^{p_{(2)}} T(v)$.

LEMMA 2. *If the Poincaré duality space K^n is also a homology manifold at the prime 2, then $p_{(2)}:\mathscr{W}_{(2)}(v) \to T(v)$ admits a section.*

Lemma 2 is a consequence of straightforward geometric facts concerning homology manifolds with coefficients, namely, that "general position theorems" of the right sort hold for these objects.

Now let G be the direct sum of countably many copies of Z_2.

LEMMA 3. *For the map $f:\mathscr{W}(v) \to \mathscr{W}_2(v)$, $\pi_i(f) = 0$, if $i \geqq 5$, $\neq 4k$. If $i = 4k \geqq 8$, then $\pi_i(f) = G$.*

Lemma 3 is an abbreviation of Theorem A below. The main consequences are

THEOREM 1. *Let M^n be a 4-connected Poincaré duality space which is a homology n-manifold at the prime 2. Then M^n has the homotopy type of a PL manifold provided a sequence of obstructions in $H^{4k}(M^n, G)$ vanish for all k such that $4k < n$.*

In reality, these obstructions are to be thought of as the Thom isomorphism images of the obstructions to lifting the section $s_{(2)}: T(v) \to \mathscr{W}_{(2)}(v)$ to a section $s: T(v) \to \mathscr{W}(v)$. Thus Theorem 1 is almost obvious by virtue of Lemmas 1, 2, 3. We only remark that if $n = 4k$, we do not need to worry

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