

SURFACES WITH A PARALLEL ISOPERIMETRIC SECTION

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This announcement is a continuation of Chen [1] (also, Yau [3]). We shall present additional theorems relating surfaces in a space form with a parallel normal section.

Let M be a surface in an m -dimensional Riemannian manifold R^m with the induced normal connection D . For a unit normal section ξ on M (that is, a unit normal vector field of M in R^m), let A_ξ be the second fundamental tensor with respect to ξ ; if we have $D\xi = 0$ identically, then ξ is called a *parallel section*; if the trace of A_ξ is constant (respectively, zero), then ξ is called an *isoperimetric section* (respectively, *minimal section*) on M ; if the determinant of A_ξ is nowhere zero, then ξ is called a *nondegenerate section*; if A_ξ vanishes identically, then ξ is called a *geodesic section*; and if A_ξ is not proportional to the identity transformation everywhere, then ξ is called a *umbilical-free section*.

THEOREM 1. *Let M be a closed surface in an m -dimensional Riemannian manifold R^m of constant sectional curvature such that the Gaussian curvature of M does not change its sign. If there exists a parallel umbilical-free isoperimetric section on M , then M is flat.*

THEOREM 2. *Let M be a closed surface of a 4-dimensional Riemannian manifold R^4 of constant sectional curvature $c \leq 0$ such that the Gaussian curvature of M does not change its sign. If there exists a parallel nondegenerate minimal section on M , then the mean curvature vector of M is parallel.*

THEOREM 3. *Let M be a surface in an m -dimensional simply-connected complete Riemannian manifold R^m of constant sectional curvature c such that the Gaussian curvature of M is constant. If there exists a parallel isoperimetric section on M , then either M is contained in a (small or great) hypersphere of R^m or M is flat.*

THEOREM 4. *Let M be a surface in a 4-dimensional simply-connected complete Riemannian manifold R^4 of constant sectional curvature $c \leq 0$ such that the Gaussian curvature of M is constant. If there exists a parallel minimal section on M , then either M is contained in a great hypersphere of R^4 or the mean curvature vector H of M is parallel and M is flat.*

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