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EQUILIBRIUM POSITIONS FOR EQUALLY CHARGED PARTICLES ON A SURFACE[1]

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ABSTRACT. This paper gives a lower bound for the number of equilibrium positions of two or three equally charged particles on an imbedded surface in Euclidean n-space.

Let $f:M \to E^n$ be a C^k ($k \geqq 2$) imbedding of a closed orientable surface into Euclidean n-space which is generic in a certain sense. This paper announces results on the lower bounds for the number of equilibrium positions of two or three equally charged particles on $f(M)$ and indicates, thereby, the manner in which the general case can be studied. For simplicity all charges are assumed to be $+1$.

1. **The 2 particle case.** The imbedding $f:M \to E^n$ is said to be *V-generic* (*potential-generic*) if the function $V_f : M \times M - D \to \mathrm{R}$ defined on $M \times M$ outside of the diagonal D by

$$V_f(x, y) = 1/\|f(x) - f(y)\|$$

satisfies the property that on $M \times M - D$ all its critical points are nondegenerate. (Any C^k ($k \geqq 2$) imbedding of M satisfies the property that there exists a real number N such that, if $V_f(x, y) \geqq N$, (x, y) cannot be a critical point of V_f.)

V_f can be easily recognized to be the potential of two unit charges on $f(M)$, so that the critical points of V_f are in fact the equilibrium positions. To compute the lower bound for the number of such positions, one observes that on $M \times M - D$, the critical points of V_f are the same as those of the function V_f^{-2}, that is, the function which assigns to (x, y) the number $\|f(x) - f(y)\|^2$. One may then apply the work of [**1**] to obtain

THEOREM 1. *Let $f:M \to E^n$ be a V-generic imbedding of a surface of genus g into E^n. Then the lower bound for the number of equilibrium positions of two equally charged particles on $f(M)$ is $2g^2 + 3g + 3$.*

2. **The 3 particle case.** The 3 particle case is exceedingly more difficult because of the homology theory involved and thereby gives an indication of the difficulty of the general case.

Consider the triple cartesian product of M with itself, $M \times M \times M$,

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