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SOME L GROUPS OF FINITE GROUPS

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If π is a finite group, define the modified Whitehead group $\mathrm{WH}'(\pi)$ to be the quotient of $\mathrm{Im}(K_1(\boldsymbol{Z}\pi) \to K_1(\boldsymbol{Q}\pi))$ (the group of reduced norms of invertible matrices over $\boldsymbol{Z}\pi$) by the classes of $\pm g$, $g \in \pi$. Using classes in this, we have a concept of 'near-simple' homotopy equivalence, and a family of surgery obstruction groups, which we denote in this paper by $L_n(\pi)$.

Roughly speaking, $L_0(\pi)$ (resp. $L_2(\pi)$) is the Grothendieck group of nonsingular hermitian (resp. skew hermitian) forms over the group ring $\boldsymbol{Z}\pi$, with involution defined by $g \mapsto w(g)g^{-1}$ $(g \in \pi)$ for some homomorphism $w:\pi \to \{\pm 1\}$; $L_1(\pi)$ (resp. $L_3(\pi)$) is the commutator quotient group of the (stable) unitary group of such forms. The precise definition is given in [**9**] or (better) [**10**]. The 'orientable' case π^+ is when w is trivial.

The object of this note is to announce the following calculations. For any abelian group G, we write ${}_2G$ and G_2 for the kernel and cokernel of $2:G \to G$.

(i) π *of odd order.* Write $R(\pi)$ for the complex representation ring of π, $\bar{x}$ for the complex conjugate of x.

$L_{2k+1}(\pi) = 0$. *The signature map on* $L_{2k}(\pi)$ *has kernel* 0 (k *even*), $\boldsymbol{Z}_2$ (k *odd*), *and image* $\{4(x + (-1)^k\bar{x}) : x \in R(\pi)\}$.

(ii) π *abelian.* Write N for the order of π, r for the 2-rank, s for the number of direct summands of order 2.

Special case. For some $x \in \pi, x^2 = 1$ *and* $w(x) = -1$. $L_n(\pi) \cong L_n(\boldsymbol{Z}_2^-) \oplus E$, *where E is an elementary 2-group of rank* $(N/2 - N/2^r - r + 1)$. $L_n(\boldsymbol{Z}_2^-) = 0$ (n *odd*) $= \boldsymbol{Z}_2$ (n *even*).

General case. There is no such x. The image of the signature map on $L_n(\pi)$ *is as in* (i) *for n even, π orientable, and* 0 *otherwise. The kernel has exponent* 2 *and rank*

$$\begin{array}{ll} 2^r - 1 - r - \binom{s}{2} & n \equiv 0, 1(4), \\ 1, & n \equiv 2(4), \\ 2^r - 1, & n \equiv 3(4)\ \textit{orientable}, \end{array}$$

exponent 2 *or* 4 *and order* $2^{(2^r + 2^{r-1} - 1)}$ *in the other case.*

(iii) π *dihedral of order* $2p$ (p *an odd prime*). Let K_p denote the maximal

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