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SYSTEMS OF QUADRATICALLY COUPLED DIFFERENTIAL EQUATIONS WHICH CAN BE REDUCED TO LINEAR SYSTEMS

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1. **Introduction.** Systems of ordinary differential equations with quadratic coupling have been used to model growth processes which occur in a number of otherwise unrelated physical applications (cf. [**1**], [**2**], [**3**]). Explicit solutions for the initial value problem have been obtained in certain cases when the coupling coefficients have been appropriately specialized (cf. [**2**], [**3**], [**4**]). This paper will consider a somewhat more general class of quadratically coupled systems for which the initial value problem can be reduced to that of a linear system.

2. **Conditions for the reduction.** The most general system of quadratically coupled differential equations over the complex field can be expressed in the form

$$\dot{x}^i + \sum_{j,k=1}^{n} \Gamma^i_{jk} x^j x^k + \sum_{j=1}^{n} A^i_j x^j = b^i, \qquad i = 1, \ldots, n. \tag{1}$$

If the coefficients in (1) are constant and satisfy the relations

$$\sum_{j=1}^{n} \Gamma^i_{jk} \Gamma^j_{lm} = \sum_{j=1}^{n} \Gamma^i_{lj} \Gamma^j_{mk}, \tag{2a}$$

and either

$$A^i_j = \sum_{k=1}^{n} \Gamma^i_{jk} a^k \tag{2b}$$

or

$$A^i_j = \sum_{k=1}^{n} \Gamma^i_{kj} a^k, \tag{2c}$$

where the a^k are the components of some constant vector a, then the solution to the initial value problem for (1) can be reduced to that for a linear system with constant coefficients.

The requirement (2a) is the necessary and sufficient condition that the Γ^i_{jk} be the structure constants for an n dimensional algebra (cf. [**5**]). In particular, the n matrices Γ_k, $k = 1, \ldots, n$, whose elements are Γ^i_{jk}, them-

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