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WEAKLY CONTINUOUS ACCRETIVE OPERATORS

BY W. E. FITZGIBBON

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We shall be concerned with the autonomous differential equation

$$(1.1) \qquad u'(t) + Au(t) = 0, \qquad u(0) = x,$$

where A is a weakly continuous possibly nonlinear operator mapping a reflexive Banach space X to itself. Recently S. Chow and J. D. Schuur [**2**] have considered existence theory for ordinary differential equations involving weakly continuous operators on separable, reflexive Banach spaces.

We now make clear our notion of strong solutions to (1.1).

DEFINITION 1.2. A function $u:[0, T) \to X$ is said to be a *strong solution* to the Cauchy problem

$$u'(t) + Au(t) = 0, \qquad u(0) = x,$$

provided that u is Lipschitz continuous on each compact subset of $[0, T)$, $u(0) = x$, u is strongly differentiable almost everywhere and $u'(t) + Au(t) = 0$ for a.e. $t \in [0, T)$.

By employing a variant of the Peano method we provide local solution to (1.1).

LEMMA 1.3. *Let X be a reflexive Banach space and suppose that A is a weakly continuous operator with $D(A) = X$. Then there is a finite interval $[0, T)$ such that the Cauchy problem* (1.1) *has a strong solution on* $[0, T)$.

DEFINITION 1.4. An operator A is said to be *accretive* provided that $\|x + \lambda Ax - (y + \lambda Ay)\| \geqq \|x - y\|$ for all $\lambda \geqq 0$ and $x, y \in D(A)$. T. Kato [**5**] has shown that this definition is equivalent to the statement that $\mathrm{Re}(Ax - Ay, f) \geqq 0$ for some $f \in F(x - y)$ where F is the duality map from X to X^*.

If we require that the operator A be accretive we are able to extend the local solution of Lemma 1.3 to a global solution.

THEOREM 1.5. *Let X be a reflexive Banach space and suppose that A is a weakly continuous accretive operator with $D(A) = X$. Then the Cauchy problem* (1.1) *has a unique strong global solution on* $[0, \infty)$.

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