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A NOTE ON ALMOST PERIODIC SOLUTIONS OF LINEAR PARTIAL DIFFERENTIAL EQUATIONS

BY GEORGE R. SELL[1]

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In this note we will present two results concerning the question of existence of almost periodic solutions of the system of linear partial differential equations

$$(1) \qquad \sum_{j=1}^{n} L_{ij}u_j = f_i, \qquad 1 \leqq i \leqq n,$$

on R^m, where L_i is an arbitrary linear partial differential operator on R^m given by

$$L_{ij} = \sum_{\alpha} a_{\alpha ij} D^{\alpha}$$

and the summation is finite. (We use the standard notation for partial differential operators, cf. [**2**] for example.) It will be more convenient to write the system (1) in the form

$$(2) \qquad Lu = f,$$

where u and f are now viewed as mappings of R^m into R^n. The order k of L is defined to be the maximum of the orders of the L_i.

We will assume that the coefficients $a_{\alpha ij}$ and f_i are continuous and almost periodic functions of $t = (t^1, \ldots, t^m)$ in R^m. Recall that g is an *almost periodic* function of t in R^m if, for every sequence $\beta' = \{\beta'_n\}$ in R^m, there is a subsequence $\beta = \{\beta_n\}$ such that $\lim g(t + \beta_n)$ converges uniformly for t in R^m. This notion of almost periodicity, which is due to Bochner for the case $R^m = R^1$, is equivalent to the Bohr concept of almost periodicity, which is defined in terms of a relatively dense set of translation numbers.

We define the *hull* $H(L,f)$ to be the collection of all linear partial differential equations $L^*u = f^*$ where the coefficients $a^*_{\alpha ij}$ and f_i^* are related to $a_{\alpha ij}$ and f_i by

$$(3) \qquad \lim a_{\alpha ij}(t + \beta_n) = a^*_{\alpha ij}(t) \quad \text{and} \quad \lim f_i(t + \beta_n) = f_i^*(t), \qquad t \in R^x,$$

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