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ON THE POLYGONAL CONNECTIVITY OF POLYHEDRA AND THE CLOSURES OF OPEN CONNECTED SETS

BY NICK M. STAVRAKAS

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ABSTRACT. Generalizations of Tietze's theorem on convex sets are given in restricted settings. For example *Theorem*: Let $S \subset R^d$ be a connected polyhedron locally homeomorphic to the closed d ball. If the points of local nonconvexity of S are decomposable into n convex sets, then S is $n + 1$ polygonally connected.

1. **Introduction.** F. A. Valentine in [6] proves the following results.

THEOREM 1. *Let S be a closed connected subset of R^d which has at most n points of local nonconvexity. Then S is $n + 1$ polygonally connected.*

THEOREM 2. *Let S be a closed connected subset of R^d. Suppose that the points of local nonconvexity of S are decomposable into n convex sets. Then S is $2n + 1$ polygonally connected.*

These results have been extended by a number of authors, but always with stronger hypothesis. See [1] and [3]. Utilizing a minimal arc technique, new proofs of Theorems 1 and 2 are given in [4], and a new characterization of the convex kernel in [5]. Guay and Kay in [2] give a new proof of Tietze's theorem using minimal arcs and the concept of m-convexity.

Valentine in [6] poses the problems of improving Theorem 2 in the cases where S is the closure of open connected set in R^d or a polyhedron in R^d. The purpose of the present paper is to announce such improvements for particular types of polyhedra and open connected sets.

2. **The improvement for the closures of open connected sets.** If A is a set, $\bar{A}$ and $L(A)$ denote the closure of A and the points of local nonconvexity of A, respectively.

DEFINITION 1. Let S be an open connected subset of R^d. We say S is locally convex connected provided, given $x \in \bar{S}$ and an open set N_x^* about x, there exists an open convex set N_x about x such that

1. $N_x \subset N_x^*$,
2. $N_x \cap S$ is connected,
3. $L(\overline{N_x \cap S}) = \bar{N}_x \cap L(\bar{S})$.

Our improvement is the following theorem.

THEOREM 3. *Let S be an open connected subset of R^d which is locally*

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