FIBERINGS OF MANIFOLDS AND TRANSVERSALITY

BY NORMAN LEVITT

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1. **Introduction and statement of results.** This paper, intended as a pendant to some earlier work of the author and J. Morgan [3] (and also of Quinn [4] and Jones [1]), deals with the following question: Given a manifold M^n which is homotopically equivalent to the total space of a fibration with a manifold K^k as fiber, when may M^n itself be fibered by K^k? (Note that if $M^n \simeq U$, and U is the total space of a K^k-bundle over a finite complex, then we may as well assume that U is a manifold with boundary and that it fibers over a manifold with boundary.)

We give sufficient conditions that this may be done, assuming that the fiber K^k is 3-connected, M^n is 4-connected, and $n - k$ is odd. With an additional assumption on the structural group of the bundle, these sufficient conditions are quite obviously necessary, thus making the theorem that much more reasonable.

First we need some definitions.

Let V^n be a PL submanifold of a PL manifold U^{n+r} (with PL normal bundle). Let W be a tubular neighborhood of V, and let T be a triangulation of U as a simplicial complex. We say that T is in general position with respect to V^n iff every simplex σ of T is in general position with respect to V^n, i.e., meets V^n in a manifold with boundary of codimension r in σ. Say that T is in general position with respect to V^n, respecting the tube W, if T is in general position with respect to V^n and for each simplex σ of $T, \sigma \cap W$ is a tubular neighborhood of $\sigma \cap V^n$.

If U is a manifold with boundary, define a *shrinking* of U as a codimension-zero submanifold $U' \subset U$ such that $U = U' \cup \partial U' \times I$ (where $\partial U' \times 0$ is identified with $\partial U'$).

Hereafter, all manifolds, bundles, etc. are to be PL. Since we will be working in high codimension, we will not have to worry about distinctions between PL, $\widetilde{\text{PL}}$, etc.

Let M^n be 4-connected, K^k 3-connected, $n - k$ odd, ≥ 5.

THEOREM A. *Sufficient conditions that M^n fibers over a manifold V^{n-k} with fiber K^k are the following:*

(i) *There is an r-dimensional PL-bundle η over M^n so that W^{n+r}, the total space of the disc bundle, fibers over the manifold with boundary Z^{n-k+r} with fiber K^k.*