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### ON A QUESTION OF DOUGLAS AND FILLMORE

BY JAMES A. DEDDENS<sup>1</sup> AND JOSEPH G. STAMPFLI<sup>2</sup>

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Let  $\mathcal{K}$  denote the ideal of compact operators in  $\mathcal{B}(H)$ , the bounded linear operators on a Hilbert space  $H$ , and let  $\nu$  denote the canonical homomorphism from  $\mathcal{B}(H)$  onto the Calkin algebra  $\mathcal{B}(H)/\mathcal{K}$ . Brown, Douglas, and Fillmore [2], in an elegant and interesting paper, showed that if  $\nu(T)$  is normal, if  $\sigma_e(T) \equiv \sigma(\nu(T))$  is homeomorphic to a finite graph, and if the Fredholm index  $i(T - \lambda) = 0$  for  $\lambda$  in the holes of  $\sigma_e(T)$ , then  $T$  is the sum of a normal operator and a compact operator. It thus becomes natural to ask whether a similar analysis can be carried out when  $\sigma_e(T)$  has positive area. Let us consider the simplest case. If  $\sigma_e(T) = \Delta$ , the closed unit disc, and if  $\nu(T)$  is normal, is  $T$  of the form normal plus compact? To indicate the limited scope of our knowledge in this area it is not even known whether the operator  $S \oplus M$  is normal plus compact, where  $S$  is the unilateral shift and  $M$  is multiplication by  $z$  on  $L^2(\Delta, dm)$ . In fact R. G. Douglas [4, p. 62] raises this specific question, which had earlier been broached by P. A. Fillmore.

In this note we will give an affirmative answer to the Douglas–Fillmore question. Unfortunately, the techniques employed here do not shed much light on the general problem.

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