BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 79, Number 1, January 1973

EQUIVARIANT SINGULAR HOMOLOGY AND COHOMOLOGY

BY SÖREN ILLMAN

Communicated by W. Browder, July 3, 1972

Let G be a topological group. By a G-space X we mean a topological space X together with a left action of G on X. Under the assumption that G is a compact Lie group, a discrete group or an abelian locally compact group, we constructed in [**5**] (see also [**6**]) an equivariant homology and cohomology theory, defined on the category of all G-pairs and G-maps, which both satisfy all seven equivariant Eilenberg-Steenrod axioms and which have the given covariant coefficient system k and contravariant coefficient system m, respectively, as coefficients. We shall here describe a simpler construction of such equivariant homology and cohomology theories which also is valid for an arbitrary topological group G. This simpler construction is described in [**6**] although main use is still made of the original more complicated construction. We shall also discuss some further properties of these equivariant homology and cohomology theories.

For actions by discrete groups equivariant cohomology and homology theories of this type exist before (see G. Bredon [**1**], [**2**] and Th. Bröcker [**3**]).

I have been informed by A. Wasserman that S. Willson has also observed that my original construction can be modified to give the simpler one described below.

1. **Equivariant singular theory.** In the following G denotes an arbitrary topological group. Let R be a ring with unit. By an R-module we mean a unitary left R-module.

DEFINITION 1. A family $\mathscr{F}$ of subgroups of G is called an orbit type family for G if the following is true: if $H \in \mathscr{F}$ and H' is conjugate to H, then $H' \in \mathscr{F}$.

Thus, the family of all closed subgroups and the family of all finite subgroups of G are examples of orbit type families for G. A more special example is the following. Let $G = O(n)$ and let $\mathscr{F}$ be the family of all subgroups conjugate to $O(m)$ (standard imbedding) for some m, where $0 \leqq m \leqq n$.

DEFINITION 2. Let $\mathscr{F}$ be an orbit type family for G. A covariant coefficient system k for $\mathscr{F}$ over the ring R is a covariant functor from the category of

AMS (MOS) subject classifications (1970). Primary 55B99, 55B25, 57E99.