BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 79, Number 1, January 1973

ON HOLOMORPHIC FAMILIES OF POINTED RIEMANN SURFACES

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Communicated by F. W. Gehring, June 26, 1972

According to a theorem of A. Grothendieck [4] the Teichmüller space of a closed Riemann surface of genus $p \geqq 2$ is the universal parameter space for holomorphic families of marked Riemann surfaces of genus p. In this note we offer a corresponding description for every finite-dimensional Teichmüller space $T(p, n)$ and discuss the universal families $\pi: V(p, n) \to T(p, n)$. Detailed proofs will be given elsewhere.

1. **The space** $T(p, n)$. Let X be the smooth (C^∞) oriented closed surface of genus $p \geqq 0$, and let $x_1, x_2, \ldots$ be a sequence of distinct points on X. Set $X_0 = X$, $X_n = X \backslash \{x_1, \ldots, x_n\}$, $n \geqq 1$. Let $\mathrm{Diff}^+ X$ be the group of orientation preserving diffeomorphisms of X, with the C^∞ topology. We define the subgroups

$$\mathrm{Diff}^+ (X, n) = \{f \in \mathrm{Diff}^+ X; f(X_n) = X_n\},$$

$$G_n = \text{the path component of the identity in } \mathrm{Diff}^+ (X, n).$$

Next we form the space M of smooth conformal structures (= complex structures) on X, again with C^∞ topology. $\mathrm{Diff}^+ X$ acts on M from the right by pullback. If the inequality

$$2p - 2 + n > 0 \tag{1}$$

holds, then the group G_n acts freely, continuously, and properly (see [3]) with local sections, and we have a principal G_n-fibre bundle. The base space M/G_n of this bundle is, by definition, the Teichmüller space $T(p, n)$. It is well known that $T(p, n)$ has a natural complex structure and can be imbedded in C^d as a bounded open contractible domain of holomorphy [2], $d = 3p - 3 + n$.

2. n**-pointed families.** Suppose the integers $p, n \geqq 0$ satisfy (1). An n-pointed family (of closed Riemann surfaces of genus p) consists of a pair of complex manifolds V and B, a holomorphic map $\pi: V \to B$, and n holomorphic sections $s_j: B \to V$ such that

(i) π is a proper submersion,

AMS (*MOS*) *subject classifications* (1970). Primary 32G15, 14H15.

[1] The author is grateful to the Institut Mittag-Leffler for financial support while this research was done.