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DIFFERENTIABLE ACTIONS OF S^1 AND S^3 ON HOMOTOPY SPHERES

BY KAI WANG

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Introduction. The purpose of this note is to announce some results on free actions of S^1 and S^3 on homotopy spheres. In the following, most of the discussion of S^3 actions will be omitted since it is completely analogous to S^1 actions. Let S^1 act on $S^{2p-1} \times S^{2q-1}$ by $g(x, y) = (gx, gy)$ for $g \in S^1$ and $(x, y) \in S^{2p-1} \times S^{2q-1}$. It is always assumed that $q \leqq p$. This is a free action and let $K^{p+q,q}$ be the orbit space. Here is the motivation for this work. Let f be a diffeomorphism of $K^{p+q,q}$ which is homotopic to the identity. Let $\bar{f}$ be its covering which is an equivariant diffeomorphism of $S^{2p-1} \times S^{2q-1}$. The manifold $\Sigma(\bar{f}) = S^{2p-1} \times D^{2q} \cup_{\bar{f}} D^{2p} \times S^{2q-1}$ obtained by gluing along $S^{2p-1} \times S^{2q-1}$ via $\bar{f}$ is a homotopy sphere. $\Sigma(\bar{f})$ supports a free S^1 action defined by $g(x, y) = (gx, gy)$ where $g \in S^1$ and $(x, y) \in S^{2p-1} \times D^{2q}$ or $D^{2p} \times S^{2q-1}$. It is easy to check that this action depends only on the pseudo-isotopy class α of f and will be denoted by $(\Sigma(\alpha), S^1)$. Let $P(\alpha)$ be the orbit space. Note that $(\Sigma(\alpha), S^1)$ is a free S^1 action on homotopy $(2p + 2q - 1)$-sphere with standard characteristic $(2q - 1)$-sphere i.e. the induced action on which is linear. Let $A^{n,q}$ be the set of all free S^1 actions on homotopy $(2n - 1)$-spheres with standard characteristic $(2q - 1)$-spheres. For $q = [(n + 1)/2]$, $A^n = A^{n,q}$ is the set of all decomposable S^1 actions on homotopy $(2n - 1)$-spheres. Similarly let B^n be the set of all decomposable S^3 actions on homotopy $(4n - 1)$-spheres (see [**6**]). For $x \in A^{n,q}$, let $s_{2k}(x)$ be the splitting invariants (see [**5**]). The main result is the following:

THEOREM. *There is a natural group structure on A^n (respectively, B^n) which makes A^n (respectively, B^n) a finitely generated abelian group of which the torsion part consists of all tangential homotopy complex projective spaces (respectively, tangential homotopy quaternion projective spaces) and* rank $A^n = [(n + 1)/4] - 1$ *if n is odd or* $[(n + 1)/4]$ *if n is even (respectively,* $[n/2] - 1$*). Furthermore,* $s_{2k}: A^n \to L_{2k}(e)$ and $s_{4k}: B^n \to Z$ *are homomorphisms.*

REMARK. The computations of torsions of A^n or B^n are reduced to the computations of $[CP^{n-1}, F]$ or $[QP^{n-1}, F]$.

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