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SPACES HOMEOMORPHIC TO $(2^\alpha)_\alpha$

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Topological characterizations and Baire-category types of properties are obtained for the spaces $(2^\alpha)_\alpha$ of all infinite regular cardinals α; in particular the stability of the class of spaces homeomorphic to $(2^\alpha)_\alpha$ when taking intersections of at most α open and dense subsets of $(2^\alpha)_\alpha$ is proved (Theorems 1, 2). Our interest in studying these spaces lies with applications of these results to questions about ultrafilters (and the Stone-Čech compactification of certain spaces) (Theorems 4, 5).

Detailed proofs of the results described in this note will appear elsewhere.

By a space we mean a completely regular Hausdorff topological space. A space X is a P_α-space if the intersection of every family of less than α open subsets of X is an open subset of X; an element p of X is a P_α-point if the intersection of every family of less than α neighborhoods of p is a neighborhood of p. A family $\mathscr{B}$ of open subsets of X is an α-subbase of X if the family $\mathscr{C}$ consisting of all intersections of families of elements of $\mathscr{B}$ of cardinality less than α is a base of X. For any space X, and any infinite regular cardinal α, two P_α-spaces are defined as follows: $P_\alpha(X)$ is the (possibly empty) subspace of X consisting of all the P_α-points of X; and X_α is the space with underlying set equal to the underlying set of X and its topology defined by the requirement that the topology of X is an α-subbase for the topology of X_α.

A space X is α-compact if every open cover of X has a subcover of cardinality less than α. A regular cardinal α is weakly compact if the space $(2^\alpha)_\alpha$ is α-compact. A weakly compact cardinal is strongly inaccessible.

If $\mathscr{B}$ is a family of closed nonempty subsets of a space X, we say that X is *$\mathscr{B}$-compact* if whenever $\mathscr{C} \subset \mathscr{B}$ and $\mathscr{C}$ has the finite intersection property (i.e., every finite subset of $\mathscr{C}$ has nonempty intersection), then $\bigcap \mathscr{C} \neq \varnothing$.

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