

## STABILITY IMPLIES NORMAL AND DISC BUNDLES<sup>1</sup>

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**Introduction.** In this note we announce results concerning normal bundles, disc bundles, and Stiefel-Whitney classes in the topological category. Many of these results also hold in the piecewise linear (PL) category, but the dimensions should be restricted accordingly.

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**Normal bundles and disc bundles.** Let  $\text{TOP}_n$  be the semisimplicial (s.s.) group of topological origin-preserving homeomorphisms of  $R^n$ . Let  $\text{TOP}_{n,k}$  be the s.s. group of topological homeomorphisms of  $R^n = R^{n-k} \times R^k$  which are pointwise fixed on  $R^k$ .

In [9] Kirby and Siebenmann announced a strong stability theorem for  $\text{TOP}/O$ , i.e., if  $n \geq 5$ , the stability map

$$s_i: \pi_i(\text{TOP}_n, O_n) \rightarrow \pi_i(\text{TOP}_{n+1}, O_{n+1})$$

is an isomorphism for  $i \leq n + 1$  and an epimorphism for  $i = n + 2$ , where  $O_n$  is the s.s.  $n$ -dimensional orthogonal group. Using this result we deduce that

**THEOREM 1.**  $\pi_i(\text{TOP}_n, \text{TOP}_n(I)) = 0$  for  $i \leq n + 1, n \geq 6$ , where  $\text{TOP}_n(I)$  is the s.s. group of topological origin-preserving homeomorphisms of the unit disc in  $R^n$ .

An immediate corollary is

**COROLLARY 2.** Let  $X$  have the homotopy type of a  $k$ -dimensional CW complex. Any  $R^n$ -bundle over  $X$  contains a disc bundle if  $n \geq k - 2, n \geq 6$ . It is uniquely determined (up to isomorphism) if  $n \geq k - 1, n \geq 6$ .

In particular, every  $n$ -manifold,  $n \geq 6$ , has a tangent disc bundle.

Using the above stability result and results of Rourke and Sanderson ([12], [13]), we show that

**THEOREM 3.**  $\pi_i(\text{TOP}_{n,k}, \text{TOP}_{n-k}) = 0$  if  $i \leq n - k + j, n - k \geq 5 + j, j = 0, 1, 2$ .

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