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FUNCTION THEORETIC METHODS FOR HIGHER ORDER, ELLIPTIC EQUATIONS IN THREE INDEPENDENT VARIABLES[1]

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Introduction. S. Bergman [1] and I. N. Vekua [5] have developed function theoretic methods for treating analytic, elliptic equations in two variables. In particular, they have developed integral operator methods for the normalized, second order equation

$$\Delta u + au_x + bu_y + cu = 0, \tag{1}$$

and the fourth order equation

$$\Delta\Delta u + au_{xx} + 2bu_{xy} + cu_{yy} + du_x + eu_y + fu = 0. \tag{2}$$

Colton [2], [3] has shown recently how one may extend the results of Bergman and Vekua for the second order case when the coefficients and solution are to be dependent on three variables. Colton's method is based on some earlier work of Tjong [4].

In this note we wish to announce that one may extend the ideas used by Colton and Tjong to treat also equations of higher order, which depend on three independent variables. We remark that this is the first time a function theoretic method has been developed for a fairly general, higher order equation in three independent variables. To simplify our presentation, and because of lack of space we will announce our results merely for the case

$$\Delta\Delta u + Q(x, y, z)u = 0; \tag{3}$$

the more general case, corresponding to equations (2) and the higher order analogues may be treated in the same manner.

The generating kernels. The approach used by Bergman and Vekua has been to continue the elliptic equation into the complex domain where it is formally hyperbolic. By introducing the variables $X = x$, $Z = \frac{1}{2}[y + iz]$,

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