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# PHRAGMEN-LINDELOF THEOREMS FOR SOME NONLINEAR ELLIPTIC PARTIAL DIFFERENTIAL EQUATIONS[1]

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The term Phragmen-Lindelof theorem, when applied to an elliptic partial differential equation, refers here to a theorem of the following type:

One considers the set of all solutions of the differential equation in a semi-infinite cylinder, which satisfies homogeneous Dirichlet data on the long sides of the cylinder and certain a priori bounds throughout the cylinder. It is then concluded that the solution, or some integral norm of the solution, decays exponentially with distance from the face of the cylinder. Results of this type are given in [**1**] for various linear elliptic partial differential equations.[2] In the mathematical theory of elasticity, Saint-Venant's Principle may be regarded as a particular example of a Phragmen-Lindelof theorem (cf. [**4**], [**5**], [**6**]).

In this paper, we consider nonlinear elliptic partial differential equations in two independent variables in the strip $R:[(x,y)/0 \leqq x,\ 0 \leqq y \leqq h]$ of the form:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^m u = \sum_{0 \leqq j \leqq m} P_{|j|}(u)\, \partial^{|j|}u\, \partial^{|2m-j|}u, \tag{1}$$

where $\partial^{|j|}$ represents a partial derivative operator of the form $(\partial/\partial x)^r(\partial/\partial y)^{j-r}$ and, for every $j$, the summation includes all operators of this form. ($\partial^0 u = u$).

For such a system, it is shown that

THEOREM 1. *Consider a differential equation of type* (1) *in* $R$, *with* $P_{|j|}(u) \in C^{m+2}[-\varepsilon, \varepsilon]$. *Let* $u$ *be a solution of the equation, which satisfies homogeneous Dirichlet boundary data at* $y = 0$ *and* $y = h$ *and such that, throughout* $R$,

$$|u| < \varepsilon < 1, \tag{2a}$$

$$|\partial^{|j|}u| < 1/a^j, \qquad j = 1, 2, \ldots, m. \tag{2b}$$

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[2] Other types of P-L theorems are given in [**2**] and [**3**].