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MANIFOLDS AS OPEN BOOKS

BY H. E. WINKELNKEMPER[1]

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Introduction. We prove that all closed, simply-connected, differentiable (or p.l.) manifolds of dimension >6 and index $\tau = 0$ decompose in a certain way: as "open books", a decomposition analogous to the classical Lefschetz decomposition of nonsingular algebraic varieties. The condition $\tau = 0$ is also necessary for a manifold to be an open book, and so, in particular, we have found a simple, intrinsic, geometric equivalent of it. For any orientable 3-manifold, this decomposition had been given by Alexander in 1923, using properties of branched coverings, which do not seem to generalize to higher dimensions. The proof of our theorem is not difficult and is a natural consequence of decomposing manifolds à la Heegaard, first accomplished for a large class of high-dimensional manifolds by Smale and completed by others [**2**], [**3**], [**8**].[2]

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1. **Statement of the theorem.** Let V be a compact differentiable $(n-1)$-manifold with $\partial V \neq \emptyset$ and $h: V \to V$ a diffeomorphism, which restricts to the identity on ∂V; by forming the mapping torus V_h, which has $\partial V \times S^1$ as boundary, and identifying $(x, t) \sim (x, t')$ on ∂V_h for each $x \in \partial V$, $t, t' \in S^1$, we obtain a closed, differentiable n-manifold M, which, if we look at a piece of the image N of $\partial V \times S^1$ under the identification map, looks like an open book (Figure 1).

The fibers of V define the 'pages' and N, a closed codimension 2 submanifold is called the 'binding'. Every point $x \notin N$ lies on one and only one page and the boundary of each page coincides with N.

DEFINITION. A closed manifold is an open book if it is diffeomorphic to one of those just obtained.

Hence an open book is represented by a page V and a self-diffeomorphism $h: V \to V$, which restricts to the identity on ∂V. If $h_*: H_*(V, Z) \to H_*(V, Z)$ is the identity, we say that the open book decomposition has no monodromy.

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[2] Also, J. P. Alexander (Ph.D. Thesis, University of California, Berkeley, 1971).