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SOME INEQUALITIES FOR UNIFORMLY BOUNDED DEPENDENT VARIABLES¹

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1. Introduction. In this note, I would like to state some inequalities, with an indication of proof. I hope to publish a more detailed treatment elsewhere.

A sum of uniformly bounded variables tends to be near the sum of the conditional expectations given the past; large deviations are exponentially unlikely, as noted in §2. The inequalities give Lévy's conditional Borel-Cantelli lemmas and his strong law as corollaries. They extend inequalities of Bernstein, Chernoff, and Hoeffding [3] to the dependent case; Hoeffding has a review of the literature.

If you study a sum of uniformly bounded variables, such that each has conditional expectation 0 given the past, and the sum of the conditional variances given the past is bounded, then large deviations are exponentially unlikely, as noted in §3. This inequality can be used to prove Lévy's law of the iterated logarithm for dependent variables. It makes explicit

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