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FUNCTIONS OF SEVERAL NONCOMMUTING VARIABLES[1]

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The notions of nonsingularity, resolvent set, and spectrum, and the corresponding analytic functional calculus for n-tuples of elements of a commutative Banach algebra provide some of the deepest and most potent tools of modern analysis. A point of view one can adopt regarding this theory is as follows: The class of algebras $\mathcal{O}(U)$, for U a domain in C^n, provides a relatively small, well-understood, and nicely behaved class of topological algebras with distinguished tuples $(z_1, \ldots, z_n)$ of elements; furthermore, spectral theory and the Shilov-Arens-Calderón Theorem (cf. [**17**]) give precise information regarding which algebras $\mathcal{O}(U)$ can be mapped into a given commutative Banach algebra (or F-algebra) by a continuous homomorphism carrying $(z_1, \ldots, z_n)$ onto a specified tuple of elements. Thus, the algebras $\mathcal{O}(U)$ and tuples $(z_1, \ldots, z_n)$ provide a tractable class of models for the behavior of n-tuples of elements of a commutative topological algebra.

In [**7**] and [**8**] we showed how to extend spectral theory and the analytic functional calculus in a well-defined manner to the study of commuting n-tuples of operators on a Banach space. From this point of view, the pairs $(\mathcal{O}(U), (z_1, \ldots, z_n))$ provide models for the behavior of n-tuples of operators.

It must occur to nearly every analyst who encounters joint spectral theory to wonder whether or not there are useful notions of nonsingularity and spectrum for tuples in a noncommutative algebra or for noncommuting tuples of operators. From the point of view we have adopted regarding spectral theory, a more meaningful question is the following: Is there a reasonably small, well behaved class of pairs $(A, (z_1, \ldots, z_n))$, consisting of an algebra A and an n-tuple $(z_1, \ldots, z_n)$ of elements of A, that will serve as models for the behavior of fairly general (noncommutative) n-tuples of algebra elements or n-tuples of operators? Given such a class, the analogue of spectral theory and the functional calculus would consist of techniques for deciding which models $(A, (z_1, \ldots, z_n))$ can be mapped into a given algebra n-tuple pair $(B, (a_1, \ldots, a_n))$, or equivalently, which models

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