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EIGENFUNCTIONS OF LAPLACE OPERATORS

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Most explicit information on the eigenfunctions of a Laplace operator on a compact manifold comes from computations where a high degree of symmetry is present. In these cases, eigenspaces may be of large dimension, the zeros of the eigenfunctions are often critical points, and the eigenfunctions usually have degenerate critical points. However, these properties are all unstable under small perturbations of the metric, and are therefore rather misleading to one's intuition. From the point of view of differential topology, the best possible properties the eigenfunctions can have would be the following:

Property A. The eigenspaces are one-dimensional.

Property B. 0 is not a critical value of the eigenfunctions (so the zero or nodal set is a manifold of codimension 1).

Property C. The eigenfunctions are Morse functions (they have non-degenerate critical points).

These properties are true for a residual set of metrics on a manifold. In the same vein we also establish similiar generic properties for bifurcations, and discuss how this approach can be used to attack the problem of invariant properties of nth eigenfunctions.

The idea for this work originated in some similiar work of J. Albert on eigenfunctions [**3**]. His methods show that the use of transversality arguments can be avoided. More general theorems and more detailed proofs will appear elsewhere [**7**].

The main technical tools used are the Sard-Smale theorem and the transversality theorems which follow from it, although all the proofs can be carried out directly by exhibiting open dense sets. A map $f: H \to E$ between two Banach manifolds is Fredholm if $Df_x: T_x(H) \to T_{f(x)}(E)$ has finite-dimensional kernel and cokernel, and its index is the difference in these two dimensions. A point $y \in E$ is a regular value for f if $x \in f^{-1}(y)$ implies Df_x is onto. A set of second Baire category is a residual set, and we have used generic to describe a property which is true for a residual set.

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