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TRANSVERSALITY STRUCTURES AND P.L. STRUCTURES ON SPHERICAL FIBRATIONS

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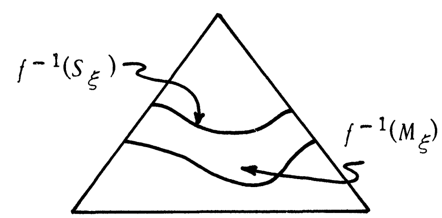
The first author in [L] introduced the notion of "Poincaré transversality" for an N-dimensional spherical fiber space, $\pi : \xi^N \to X$. If $T(\xi^N)$ is the Thom space of ξ^N, then we consider $T(\xi^N) = M_\xi \cup c(S(\xi))$ where M_ξ is the mapping cylinder of π and $S(\xi)$ is the total space of ξ^N. A map $f: \Delta^{N+i} \to T(\xi^N)$ is Poincaré transversal if f is p.l. transversal to $S(\xi) \subset T(\xi)$ with $(f^{-1}(M_\xi), f^{-1}(S(\xi)))$ a codimension 0 submanifold of Δ^{N+i} with the inclusion $f^{-1}(S(\xi)) \subset f^{-1}(M_\xi)$ the spherical fibration induced by f over $f^{-1}(M_\xi)$. This implies $f^{-1}(M_\xi)$ is a Poincaré duality space, (P.D. space), of dimension i with boundary $f^{-1}(M_\xi) \cap \partial\Delta^{i+N}$, and that $(f^{-1}(M_\xi), f^{-1}(S_\xi))$ is its normal tube.



A p.l. manifold M^j mapping by $f: M^j \to T(\xi^N)$ is Poincaré transversal to ξ^N if and only if $f|$(any simplex) is. If f is Poincaré transversal then $f^{-1}(M_\xi)$ is a P.D. space with boundary $f^{-1}(M_\xi) \cap \partial M$ and of dimension $j - N$. One of the main results of [L] is to develop a theory to study the problem of when a map $f: M^j \to T(\xi^N)$ may be shifted to be Poincaré transversal. To do this one introduces the space $W(\xi^N)$, of Poincaré transversal maps of $\Delta^i \to T(\xi^N)$ for all i. In [L] and [J] it is proved that if F_{ξ^N} denotes the homotopy theoretic fiber of $W(\xi^N) \to T(\xi^N)$, then $\pi_i(F_{\xi^N}) \cong \pi_{i-N}(G/PL)$ for $i - N \neq 1, 2,$ or 3. In fact a map of fiber spaces $\xi^N \to \zeta^N$ induces $F_{\xi^N} \to F_{\zeta^N}$ and this map is an isomorphism on π_i for $i - N \neq 1, 2,$ or 3. Also if $f: M^j \to T(\xi^N)$, then homotopying f until it is Poincaré transversal is equivalent to lifting f up to homotopy to $W(\xi^N)$. In this announcement we shall describe further results in this theory.

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