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THE NUMBER OF UNLABELLED GRAPHS WITH MANY NODES AND EDGES

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$T = T(n, q)$ is the number of graphs with n unlabelled nodes and q undirected edges, each pair of different nodes being not joined or joined by a single edge. We write $N = n(n-1)/2$, so that a graph contains at most N edges. We give here asymptotic approximations to T for all large n and q. Since $T(n, q) = T(n, N - q)$, we suppose $q \leqq N/2$ throughout.

In what follows, A, C and η denote numbers, not always the same at each occurrence. Of these, A and C are positive and independent of n and q. A number A denotes any positive number that we may choose, while C is a suitable positive number which may depend on any A present or implied. Unless we specifically state the contrary, all our statements carry the implied condition that $q > C$ and $n > C$. The O-notation refers to the passage of n and q to infinity and the constant implied is a C. An η is any number which is $O(q^{-C})$ for some C.

We write

$$B(h, k) = h!/\{k!(h-k)!\}, \qquad \Lambda_n = \Lambda(n, q) = B\{n(n-1)/2, q\}/n!,$$

$$\mu = (2q/n) - \log n, \qquad J(v) = v^{1/2}\{2(1 + \log v)\}^{-1/2},$$

$$\delta = \mu J(n), \qquad K(v) = 2\pi^{1/2}e^{-1}J(v),$$

$$\operatorname{Erf} x = 2\pi^{-1/2}\int_0^x e^{-t^2}\,dt, \qquad \lambda(x) = (1 + \operatorname{Erf} x)/2.$$

A table of $\operatorname{Erf} x$ is given in [**1**]. We write V to denote the greatest integer such that $V \log V \leqq 2q$.

Polya [**2**] proved that $T \sim \Lambda_n$ as $n, q \to \infty$, provided that $(N/2) - q < An$, and Oberschelp [**5**] weakened the condition to $(N/2) - q < Cn^{3/2}$, In [**8**], I proved the following theorem.

THEOREM 1. *The necessary and sufficient condition that $T \sim \Lambda_n$ is that $\mu \to \infty$ as $n \to \infty$. If this condition is satisfied, then $T = \Lambda_n\{1 + O(e^{-C\mu})\}$.*

Korsunov [**4**] stated the following theorem without proof.

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