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# INFINITE-DIMENSIONAL METHODS IN FINITE-DIMENSIONAL GEOMETRIC TOPOLOGY

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1. **Introduction.**[2] We use the methods of infinite-dimensional topology to derive new information about the topology of euclidean spaces and manifolds. The idea is to partition euclidean $n$-space $E^n$ into a $k$-dimensional pseudo-boundary $(0 \leqq k < n)$ and an $(n-k-1)$-dimensional pseudo-interior, and to deduce negligibility theorems analogous to those known for the pseudo-boundary and the pseudo-interior (denoted by $s$) of the Hilbert cube $I^\omega$. Since $s$ is homeomorphic to Hilbert space $l_2$, there is a sense in which we are giving the correct finite-dimensional analogues of $l_2$ (see §5).

DEFINITION. A subset $X$ of a metric space $Y$ (with metric $d$) is *strongly negligible in* $Y$ if, for each open set $U$ in $Y$ and each map $\varepsilon: U \to R^+$, there is a homeomorphism $h: Y \to Y - (X \cap U)$ fixing $Y - U$ such that $d(x, h(x)) < \varepsilon(x)$ for all $x \in U$. This is a topological property independent of $d$.

THEOREM 1.1. *$E^n$ is the union of two disjoint dense subsets $B^k$ and $P^{n-k-1}$ such that* (1) *if $n \leqq 2k+1$, any $\sigma$-compact subset of $P^{n-k-1}$ is strongly negligible in $P^{n-k-1}$, and* (2) *if $n \geqq 2k+1$, any compact subset of $B^k$ is strongly negligible in $B^k$. If $n = 2k+1$, any $k$-dimensional compactum can be embedded in $B^k$ or in $P^k$.*

NOTATION. Superscripts on spaces, e.g., $B^k$, $P^{n-k-1}$, indicate dimension.

We call $B^k$ of Theorem 1.1 *the universal $k$-dimensional pseudo-boundary of $E^n$*. It is built out of Menger universal compacta [**13**], [**17**]. (See §3.) $P^{n-k-1}$ of Theorem 1.1 is the corresponding pseudo-interior.

Another kind of $k$-dimensional pseudo-boundary in $E^n$ can be built out of polyhedra as follows.

Let $J_0$ be a rectilinear PL triangulation of $E^n$, all $n$-simplexes having the same diameter. Let $J_i$ $(i \geqq 1)$ be the $i$th barycentric subdivision of $J_0$, its $k$-skeleton being $J_i^k$. The *polyhedral $k$-dimensional pseudo-boundary of $E^n$* is $\tilde{B}_n^k = \bigcup_{i=1}^{\infty} |J_i^k|$. The corresponding pseudo-interior is $\tilde{P}_n^{n-k-1} = E^n - \tilde{B}_n^k$.

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[2] This is a summary of a paper entitled *Pseudo-boundaries and pseudo-interiors in euclidean spaces and topological manifolds.*