BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 6, November 1972

THE SECOND OBSTRUCTION FOR PSEUDO-ISOTOPIES

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Communicated by Michael Atiyah, May 18, 1972

1. **Introduction.** In this note is announced the completion of the reduction to algebra of the pseudo-isotopy problem for smooth compact manifolds of large dimension. This is to compute $\pi_0\mathscr{P}(M, \partial M)$ where, for a smooth manifold $M, \mathscr{P}(M, \partial M)$ is the group of diffeomorphisms of $M \times I$ which are the identity on $M \times \{0\} \cup \partial M \times I$, $\mathscr{P}(M, \partial M)$ being given the C^∞ topology.

THEOREM. *If M is compact, connected and of dimension at least seven, then*

$$\pi_0\mathscr{P}(M, \partial M) \approx \mathrm{Wh}_2\pi_1 M \oplus \mathrm{Wh}_1(\pi_1 M; \boldsymbol{Z}_2 \times \pi_2 M).$$

For M simply-connected this is Cerf's theorem that $\pi_0\mathscr{P}(M, \partial M) = 0$ [**2**]. The Wh_2 factor has been described by J. B. Wagoner [**7**] and, independently, by the author [**4**]. Partial results on the Wh_1 factor have appeared in [**3**].

We now define and compute $\mathrm{Wh}_1(\pi_1; \boldsymbol{Z}_2 \times \pi_2)$. Let the group π act on the abelian group Γ. Giving Γ trivial multiplication, form the group ring $\Gamma[\pi]$. This fits into a split exact sequence

$$0 \to \Gamma[\pi] \to (\Gamma \times \boldsymbol{Z})_T[\pi] \to \boldsymbol{Z}[\pi] \to 0,$$

where the twisting T is given by $\sigma(\alpha\tau) = \alpha^\sigma\sigma\tau$ for $\sigma, \tau \in \pi$, $\alpha \in \Gamma$, and α^σ denotes the action of σ on α. Then $\Gamma[\pi]$ is an ideal in $(\Gamma \times \boldsymbol{Z})_T[\pi]$ and the relative group $K_1\Gamma[\pi]$ is defined as in [**2**], [**6**]. Its elements are represented by matrices $I + A$, where A has entries in $\Gamma[\pi]$.

PROPOSITION. *$K_1\Gamma[\pi] \approx \Gamma[\pi]/(\alpha\sigma - \alpha^\tau\tau\sigma\tau^{-1})$, via $[I + A] \mapsto \mathrm{trace}\,(A)$.*

Here "(—)" denotes "additive subgroup generated by —,'.

Define $\mathrm{Wh}_1(\pi; \Gamma)$ as the cokernel of $K_1\Gamma[1] \to K_1\Gamma[\pi]$. Note that $\mathrm{Wh}_1(\pi; \Gamma)$ is unrelated to the classical Whitehead group $\mathrm{Wh}_1\pi$ since Γ was given trivial multiplication, e.g., $\mathrm{Wh}_1(\pi; \boldsymbol{Z}) \neq \mathrm{Wh}_1\pi$.

Now let $\pi = \pi_1$ and $\Gamma = \boldsymbol{Z}_2 \times \pi_2$, with the usual action of π_1 on π_2 and the trivial action on $\boldsymbol{Z}_2$ ($\boldsymbol{Z}_2 = \boldsymbol{Z}/2\boldsymbol{Z}$).

AMS 1970 *subject classifications*. Primary 57D50, 57D80; Secondary 18F25.

[1] Supported by an NSF Postdoctoral Fellowship.