

## APPLICATIONS OF THE TRANSFER TO STABLE HOMOTOPY THEORY

BY DANIEL S. KAHN<sup>1</sup> AND STEWART B. PRIDDY<sup>2</sup>

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In this note we outline the construction, properties and several applications of a transfer morphism for the generalized cohomology of finite coverings. Our principal application consists of a proof of a conjecture of M. E. Mahowald [9] and G. W. Whitehead [14]. Let  $\lambda: \Sigma^n RP^{n-1} \rightarrow S^n$  be the adjoint of  $RP^{n-1} \xrightarrow{i_n} O_n \xrightarrow{j_n} \Omega^n S^n$  where  $i_n$  represents a line  $L$  through the origin in  $R^n$  as the reflection in the hyperplane perpendicular to  $L$  and  $j_n$  represents an element of  $O_n$  as a map  $(R^n \cup \infty, \infty) \rightarrow (R^n \cup \infty, \infty)$ . Then, for  $0 < i < n - 1$ ,

$$(0.1) \quad \lambda_{n+i}: \pi_{n+i}(\Sigma^n RP^{n-1}) \rightarrow \pi_{n+i}(S^n)$$

is an epimorphism of 2-primary components (see §3).

The existence of the transfer seems to be well known [13], but we know of no published account. In §1 we outline such a construction. The essential connection between the transfer and stable homotopy theory is provided by a stable map which yields the transfer as an induced homomorphism (Proposition 1.7). As an immediate consequence, the transfer commutes with stable cohomology operations. This generalizes the same result for ordinary cohomology of groups proved by Evens [7]; similarly it gives an alternate proof of Quillen's result [12] that the localized Adams operations  $\psi^p[p^{-1}]$  commute with the transfer in  $K$ -theory. As a final application we give a stable decomposition of  $(\Omega^\infty S^\infty)_0$ . Details will appear elsewhere,

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**1. The transfer for generalized cohomology.** Let  $\pi: E \rightarrow B$  be a finite covering of degree  $N$  (for the purposes of this note we assume  $E$

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