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QUASI-ANALYTICITY AND SEMIGROUPS

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ABSTRACT. Four problems and their interrelationships are considered. These problems concern (1) quasi-analyticity conditions in terms of finite differences, (2) quasi-analyticity conditions for one-parameter semigroups of linear transformations, (3) generation (in the sense of S. Lie) of one-parameter semigroups of nonlinear transformations and (4) quasi-analyticity conditions for one-parameter semigroups of nonlinear transformations. The quasi-analyticity conditions in (2) and (4) are in terms of the degree of approximation of the identity by a semigroup. In connection with (3) an infinitesimal generator and a corresponding exponential formula are obtained without assuming differentiability.

1. This work deals with quasi-analyticity, with linear and nonlinear one-parameter semigroups of transformations and with certain relationships between these subjects. If J is a connected set of real numbers, then a collection G of functions with common domain J is said to be quasi-analytic provided no two members of G agree on an open subset of J.

Suppose H is a Banach space. If C is a subset of H, a semigroup on C is a function T with domain $[0, \infty)$ such that

(1) If $\lambda \geqq 0$, then $T(\lambda)$ is a transformation from C to C, and

(2) If $s, t \geqq 0$, then $T(t)T(s) = T(t + s)$ and $T(0)$ is the identity transformation on C.

If p is in C, then g_p denotes the function from $[0, \infty)$ to C such that $g_p(\lambda) = T(\lambda)p$ for all $\lambda \geqq 0$ (g_p is called a trajectory of T). If p is in C and f is in H^*, then the composition fg_p is denoted by $z_{p,f}$ and is called a functional of a trajectory of T. If $r \geqq 0$, then T is called r-quasi-analytic provided no two continuous functionals of trajectories of T agree on an open subset of $(r/2, \infty)$ unless they agree on all of $(r/2, \infty)$. If T is r-quasi-analytic and $r = 0$, then T will be called quasi-analytic. If g_p is continuous for all p in C, then T is called strongly continuous.

Quasi-analytic collections of real-valued functions have been studied by S. Bernstein, Carleman, Denjoy, Wiener, Beurling, de la Vallée Poussin, Mandelbrojt and others. Descriptions of Bernstein's work may be found in [3] and [37]. The reader is also referred to Carleman's book [**6**], to

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