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DEVELOPMENTS IN SCHAUDER BASIS THEORY[1]

BY C. W. McARTHUR[2]

1. **Introduction.** In the forty-four years since 1927 when J. Schauder [**101**] introduced the notion of a topological basis for a Banach space, well over two hundred papers on basis theory have been published. About one-fifth of these appeared in the twenty-three years before 1950 with the other four-fifths appearing in the twenty-one years from 1950 to the present. Surely the devastating effect of World War II is one factor in this distribution of mathematical output. Indeed, two of the originators of the abstract theory of bases, J. Schauder and S. Banach, were essentially casualties of the war, Schauder losing his life during the war and Banach dying prematurely shortly thereafter.

It will be the purpose of this paper to trace the development up to the present time of several theorems or problems introduced in 1932 by Banach [**6**], as well as to report on some developments not anticipated by Banach.

We first mention a problem considered by Fréchet and Banach which has only recently been solved and in whose solution basis theory played an important role. Which of the topological linear spaces are homeomorphic? In 1928, [**33**, pp. 94–96] Fréchet asked whether the separable Hilbert space l^2 was homeomorphic to (s), the space of all real sequences with the product topology. In 1932, [**6**, p. 233] Banach stated that Mazur had shown that (s) was not homeomorphic to l^2. It was subsequently realized that the question was still open. Indeed, in 1966 [**1**], R. D. Anderson was the first to prove that l^2 and (s) are homeomorphic. C. Bessaga and A. Pelczynski [**8**], [**13**] in the meantime had shown that "Under the conjecture that all separable infinite dimensional Banach spaces are homeomorphic to l^2, every separable infinite dimensional Fréchet space E, with $E \neq (s)$ is homeomorphic to l^2." Thus when M. I. Kadec [**53**] showed, using basis theory as a main tool, that all separable infinite dimensional Banach spaces are homeomorphic, Anderson's result provided the missing link to give the startling result: All separable infinite dimensional Fréchet spaces are homeomorphic.

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