BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 78, Number 5, September 1972

CHARACTERIZING SHAPES OF COMPACTA

BY T. A. CHAPMAN[1]

Communicated by Steve Armentrout, October 18, 1971

ABSTRACT. In this note we announce a characterization of Borsuk's concept of shape for finite-dimensional compact metric spaces. Let X, Y be compact subsets of some Euclidean space E^n such that (1) X lies in some Euclidean subspace of E^n having codimension at least $2 \dim X + 1$ and (2) Y lies in some Euclidean subspace of E^n having codimension at least $2 \dim Y + 1$. Then n can be chosen large enough (and dependent only on $\dim X$, $\dim Y$) such that the following is true: X and Y have the same shape iff $E^n \backslash X$ and $E^n \backslash Y$ are homeomorphic. We also discuss the relationship of this result to previously known characterizations of shape.

1. **Introduction.** The objective of this note is to announce a characterization of *shape* (as introduced by Borsuk in [4]) for finite-dimensional compact metric spaces in terms of the homeomorphism types of the complements under stable embeddings in large-dimensional Euclidean spaces. The precise statement is given in §4 and the details of its proof will appear elsewhere [9]. This result confirms an informal conversational conjecture of Morton Brown concerning the geometric intuition about what shape ought to mean. It and a companion earlier result of the author [6] cited in §3 below give easily stated characterizations of shape. They also put the study of shape in the point-set (homeomorphism) domain as distinct from the homotopy domain of the definition of Borsuk and of the characterization of Mardesic and Segal cited in §3 below.

In §2 we make some intuitive comments concerning shapes of compact metric spaces (compacta) without actually giving any definitions. This section is intended only for those readers not familiar with shape theory. In §3 we cite some other instances in which shapes of compacta have been characterized by using approaches quite different from that of Borsuk who views shape as a generalization of homotopy type to non-ANR's (metric). The notation

$$\mathrm{Sh}(X) = \mathrm{Sh}(Y)$$

will be used to indicate that compacta X and Y have the same shape.

2. **Intuitive shape theory.** The term *shape theory* actually refers to an entire category of objects (compacta) and morphisms (which we shall not specify), but we are only interested here in the equivalence relation

AMS 1969 *subject classifications.* Primary 5540; Secondary 5478.
Key words and phrases. Shape, ANR, Hilbert space, Hilbert cube.
[1] Supported in part by NSF Grant GP-28374.