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ON THE SUMMATION OF CONJUGATE FOURIER INTEGRALS OF FUNCTIONS OF SEVERAL VARIABLES

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Let K denote a homogeneous Calderón-Zygmund singular integral kernel $K(x)$ which is bounded and has mean value 0 on the unit sphere $S^{n-1} = \{x : |x| = 1\}$ of R^n and let $\hat{K}$ denote its principal valued Fourier transform (see [2]). In this note some simple propositions are stated which generalize results of V. L. Shapiro and others and can be proved in an elementary fashion. Suppose the Fourier transform of any integrable function f is defined by $\hat{f}(x) = \int f(y)e^{-ixy}\,dy$. The truncated kernel K_ε is defined by $K_\varepsilon(x) = K(x)$ if $|x| \geqq \varepsilon$ or $K_\varepsilon(x) = 0$ otherwise. For any function g not denoted by K and $\varepsilon > 0$ let $g_\varepsilon(x) = \varepsilon^{-n}g(\varepsilon^{-1}x)$, $g^\varepsilon(x) = g(\varepsilon x)$.

PROPOSITION 1. *Suppose $K(x) = \Omega(x)|x|^{-n}$ where Ω is homogeneous of degree 0, has mean value 0 on S^{n-1} and its modulus of continuity defined by*

$$\omega(t) = \sup\{|\Omega(\xi + h) - \Omega(\xi)| : |\xi| = 1, |h| \leqq t\}$$

satisfies the Dini condition $\int_0^1 \omega(t)\,dt/t < \infty$.

Furthermore suppose the integrable function φ is such that

$$\psi_0(x) = \int_{1 \leqq |x-y| \leqq |x|/2} K(x-y)\varphi(y)\,dy, \tag{1}$$

$$\psi_1(x) = \int_{|y| \geqq |x|} \varphi(y)\,dy, \tag{2}$$

$$\psi_2(x) = \int_{|y| \leqq 1} |y|^{-n}\,|\varphi(x-y) - \varphi(x)|\,dy \tag{3}$$

all satisfy

$$\int_0^\infty \sup_{t \leqq |x| \leqq 2t} |\psi_j(x)|\,t^{n-1}\,dt < \infty. \tag{4}$$

Finally suppose $\hat{\varphi} \in L^1$. Then, for $f \in L^1$ and $\int \varphi(x)\,dx = m$,

$$\lim_{\varepsilon \to 0}\left[(2\pi)^{-n}\int \hat{f}(y)\hat{K}(y)e^{ixy}\hat{\varphi}(\varepsilon y)\,dy - mK_\varepsilon * f(x)\right] = 0 \tag{5}$$

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