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FOLIATIONS AND THE PROPAGATION OF ZEROES OF SOLUTIONS OF PARTIAL DIFFERENTIAL EQUATIONS

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1. **Introduction.** Let $P(x, D)$ be a linear partial differential operator of order m with complex-valued coefficients defined and analytic in an open connected set Ω in R^n,

$$P(x, D) = \sum_{|\alpha| \leq m} a^\alpha(x) D^\alpha, \tag{1}$$

with the usual notation. The principal part $P_m(x, D)$ is the homogeneous part of $P(x, D)$ of order m. At a fixed point $x \in \Omega$, the (real) zeroes of $P_m(x, \xi)$ form a cone in R^n which is called the (real) characteristic cone of $P(x, D)$ at x. We will denote by $\mathscr{A}$ the ring of real-valued analytic functions in Ω.

In this paper we consider partial differential operators having the following property: There exist r analytic vector fields in Ω,

$$A_j = \sum_{i=1}^{n} a_j^i D_i, \qquad j = 1, \ldots, r, \tag{2}$$

with $a_j^i \in \mathscr{A}$, $i = 1, \ldots, n$, $j = 1, \ldots, r$, such that at each point of Ω the characteristic cone of $P(x, D)$ is orthogonal to every A_j. More precisely, we assume that, for every $x \in \Omega$,

$$P_m(x, \xi) = 0, \quad \xi \in R^n \Rightarrow \sum_{i=1}^{n} a_j^i(x) \xi_i = 0, \qquad j = 1, \ldots, r. \tag{3}$$

We will denote by $\mathscr{L}(A_1, \ldots, A_r)$ the Lie algebra generated by $A_1, \ldots, A_r$, i.e. the smallest set of analytic vector fields in Ω which is closed under the operations of taking brackets and linear combinations with coefficients in $\mathscr{A}$.

According to a theorem of Nagano [1], the Lie algebra $\mathscr{L}(A_1, \ldots, A_r)$ defines a unique partition of Ω into maximal integral manifolds of $\mathscr{L}(A_1, \ldots, A_r)$, that is, Ω is the disjoint union of maximal integral manifolds of $\mathscr{L}(A_1, \ldots, A_r)$. This partition is called a foliation and each maximal integral manifold is called a leaf of the foliation.

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