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ON $\mathrm{Ext}_R^1(A, R)$ FOR TORSION-FREE A

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For any Dedekind domain R (or more generally a Prüfer domain) the modules of the form $\mathrm{Ext}_R^1(A, R)$, A being a torsion-free R-module, coincide with those of the form $\varprojlim^{(1)} F_\alpha$ for projective systems of finitely generated free R-modules F_α (cf. [1], [5]) and appear in various topological contexts.

It is the purpose of this note to outline some results concerning the structure of the above class of modules, in particular, in the case where A is the quotient field Q of R. As a by-product we get a negative solution of an analogon of the following (open) problem [4]: If Z is the ring of integers, can $\mathrm{Ext}_Z^1(A, Z)$ ever be a nonzero torsion group? In fact, we shall obtain a class of principal ideal domains R such that $\mathrm{Ext}_R^1(A, R) \simeq Q/R$ for a suitable torsion-free R-module A.

Generally, for any integral domain R with quotient field Q $\mathrm{Ext}_R^1(Q, R)$ may be viewed as a vector space over Q and thus

$$(*) \qquad \mathrm{Ext}_R^1(Q, R) \simeq Q^{(d)},$$

for some finite or infinite cardinal number d.

THEOREM 1. *Let R run through the class of all Noetherian domains of Krull dimension* 1 *which are analytically unramified in at least one maximal ideal. Then the cardinal numbers d which appear in* (*) *are*

(i) *any infinite cardinal number,*

(ii) *among the finite cardinal numbers exactly those of the form $p^t - 1$, p being a prime number and t an integer $\geqq 0$.*

When d is finite, and $\neq 0$ R is necessarily local and of prime characteristic. Moreover, any d from (i) *and* (ii) *can occur for principal ideal domains, in fact even for valuation rings.*

Before we sketch the main steps in the proof we show how we by Theorem 1 (for $p = 2$, $t = 1$) get a principal ideal domain R such that $\mathrm{Ext}_R^1(A, R) \simeq Q/R$ for some torsion-free R-module A.

PROPOSITION 2. *If R is a discrete valuation ring for which $\mathrm{Ext}_R^1(Q, R) \simeq Q$, then there exists a torsion-free R-module A such that $\mathrm{Ext}_R^1(A, R) \simeq Q/R$.*

PROOF. Since $\mathrm{Ext}_R^1(Q, R) \neq 0, R$ is not a complete valuation ring. By [2, Theorem 19], there exists an indecomposable torsion-free R-module A

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