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FOURIER COEFFICIENTS OF CERTAIN EISENSTEIN SERIES[1]

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Let K be a field of characteristic $\neq 2, 3$ and let $\mathfrak{J}_K$ be the exceptional Jordan algebra of dimension 27 consisting of hermitian 3×3 matrices with entries in the Cayley-Dickson algebra $\mathfrak{C}_K$. The product $X \circ Y$ in $\mathfrak{J}$ is $\frac{1}{2}(XY + YX)$, where XY is the matrix product. In [3], there are defined a norm (det) and a trace (tr) on $\mathfrak{J}$. Let $(\ ,\ ,\ )$ be the symmetric trilinear form on $\mathfrak{J} \times \mathfrak{J} \times \mathfrak{J}$ such that $(A, A, A) = \det(A)$, and define a bilinear map $\mathfrak{J} \times \mathfrak{J} \to \mathfrak{J}$, which takes (A, B) to $A \times B$, by requiring that $(A \times B, C) = 3(A, B, C)$ for each $C \in \mathfrak{J}$, where $(X, Y) = \operatorname{tr}(X \circ Y)$. Then $A \times A$ plays the role of the matrix adjoint of A, and the notions just introduced can be used to define the rank of each element $A \in \mathfrak{J}$. We denote this by $\operatorname{rk}(A)$. In particular, $\operatorname{rk}(A) = 3$ if and only if $\det(A) \neq 0$. Let $\mathfrak{k}_j = \{A \in \mathfrak{J}_R : \operatorname{rk}(A) = j\}$. The tube domain associated to $\mathfrak{J}$ is

$$\mathfrak{T} = \{Z = X + iY \in \mathfrak{J}_C : Y \in \mathfrak{k}_3^+\},$$

where $\mathfrak{k}_j^+ = \{Y \in \mathfrak{k}_j : Y = X^2 \text{ for some } X \in \mathfrak{J}_R\}$.

The group of holomorphic automorphisms of $\mathfrak{T}$ is isogenous to a certain algebraic Q-group which is of type E_7. Baily [1] has defined an arithmetic subgroup Γ of G_Q which is a unicuspidal subgroup of G and a maximal discrete subgroup of G_R. Let $J(Z, \gamma)$ be the functional determinant of γ at $Z, Z \in \mathfrak{T}$. Let Γ_0 be the subgroup of Γ which stabilizes a certain zero-dimensional rational boundary component $\mathfrak{T}_0^\infty$ of $\mathfrak{T}$, as in [1, §7]. We let

$$E_g(Z) = \sum_{\gamma \in \Gamma/\Gamma_0} J(Z, \gamma)^{g/18},$$

where $g \equiv 0 \pmod{36}$ and $g > 19$. Then the Eisenstein series E_g is an automorphic form of weight $g/18$ with respect to the group Γ and the factor of automorphy J. It has an absolutely convergent Fourier expansion

$$E_g(Z) = \sum_{T \in \Lambda^+} a_g(T) e^{2\pi i (T, Z)},$$

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[1] This paper describes a portion of the author's doctoral thesis written under the direction of Professor Walter L. Baily, Jr. at the University of Chicago.