

## COMPACTIFICATIONS OF $C^2$

BY JAMES A. MORROW

Communicated by S. S. Chern, March 13, 1972

**1. Introduction.** By a *compactification* of  $C^2$  we mean a nonsingular compact complex manifold  $M$  of complex dimension 2 which contains a nonempty nowhere dense closed analytic subset  $A$  such that  $M - A$  is biholomorphic to  $C^2$ . It is not hard to verify that  $A$  is a compact connected one-dimensional analytic set, hence a finite union of irreducible curves. By blowing up certain points of  $A$  we may assume that  $A$  has the following properties.

- (1)  $A = \bigcup_{i=1}^k \Gamma_i$ , where  $\Gamma_i$  is a nonsingular connected algebraic curve.
- (2)  $\Gamma_i$  intersects  $\Gamma_j$  normally (if at all).
- (3)  $\Gamma_i \cap \Gamma_j \cap \Gamma_k = \emptyset$  for any three distinct indices.
- (4) If the self-intersection  $(\Gamma_i)^2 = -1$ , then  $\Gamma_i$  meets at least three other curves  $\Gamma_j$ .

We call such a compactification a *minimal normal compactification* of  $C^2$ . The purpose of this note is to announce a list of all minimal normal compactifications of  $C^2$ . The proofs will appear elsewhere.

**2. Sketch of method.** The construction and proofs rely heavily on [3] and [4]. It is not hard to prove (see [5]) that each  $\Gamma_i \cong P^1(C)$ . A theorem of van de Ven [4] says that  $M$  is necessarily algebraic. A result of Ramanujam [3] says that the graph of  $A$  is linear. One then uses a surgical technique to find what possible selfintersection numbers the  $\Gamma_i$  can have. One step in the proof uses a theorem of Mumford [2] to compute the fundamental group of the boundary of a tubular neighborhood of  $A$ . One can then produce a list of possible graphs. One can prove that the compactifications corresponding to these graphs actually occur and are uniquely determined by the graphs. This has as a corollary the fact that all compactifications of  $C^2$  are rational, a result conjectured by van de Ven and recently proved by Kodaira [1] by different techniques.

**3. The list of graphs.** The notation is as follows. Each line represents a point of intersection and each circle ("vertex") represents a nonsingular rational curve ( $P^1(C)$ ). The number adjacent to each circle is the self-

---

AMS 1970 subject classifications. Primary 32J05, 32J15; Secondary 14J99.