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ON THE 2-SPHERES IN A 3-MANIFOLD

BY F. LAUDENBACH

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We shall give here an abstract of [2]. In this paper, the manifolds and the maps are C^∞. Let us recall some definitions:

(1) Let S and S' be two spheres in a 3-manifold V^3; S and S' are said to be homotopic if there exist two embeddings $\varphi, \varphi' : S^2 \to V$ with S and S' as images that are homotopic in the family of smooth maps $S^2 \to V$. S and S' are said to be isotopic if there exists a diffeomorphism H of V, isotopic to the identity, such that $H(S) = S'$.

(2) $h : S^2 \times [0, 1] \to V$ is said to be a homotopy of disjunction of S' from S if $h|S^2 \times \{0\}$ is an embedding with S' as image and if $h(S^2 \times \{1\}) \subset V - S$.

(3) V satisfies the Poincaré conjecture if any compact contractible 3-manifold of V is diffeomorphic to D^3.

We obtain the following results:

THEOREM I. *Let V be a 3-manifold satisfying the Poincaré conjecture and let S, S' be two spheres in V. If there exists a homotopy of disjunction of S' from S, then there exists an isotopy of disjunction.*

THEOREM II. *With the same hypotheses as in Theorem I, if S and S' are homotopic, then S and S' are isotopic.*

THEOREM III. *For any positive integer p, we shall denote $p \# S^1 \times S^2$ the connected sum of p copies of $S^1 \times S^2$. Let H be a diffeomorphism of $p \# S^1 \times S^2$ homotopic to the identity. Then H is isotopic to the identity.*

REMARK. Theorems I and II are trivial if S is null homotopic, because then S is the boundary of a ball; hence we suppose that S is not null homotopic.

THEOREM I $\Rightarrow$ THEOREM II. If S' is homotopic to S, then there exists a disjunction homotopy of S' from S. After Theorem I, there exists an isotopy of disjunction. If now S and S' are homotopic and disjoint, they bound an h-cobordism, which is trivial, since V satisfies the Poincaré conjecture.

THEOREM II $\Rightarrow$ THEOREM III. Let $\Sigma_1, \ldots, \Sigma_p$ be the p transversal spheres in the index 1 handlebodies of $p \# S^1 \times S^2$. We can, by using mainly Theorem II, reduce to the case where $H|\Sigma_1 \cup \ldots \cup \Sigma_p$ is the

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