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FIBER PRESERVING CELLULAR DECOMPOSITIONS

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1. **Introduction.** Let (Y, P, B) be a locally trivial bundle with fiber M^r $(r \neq 4)$, a compact connected metric manifold with or without boundary; furthermore, assume B is a finite-dimensional, locally compact metric space. Let $G = \{G[b] \mid b \in B\}^*$ be an upper semicontinuous (usc) decomposition of Y with each $G[b]$ a cellular decomposition of $P^{-1}(b)$ such that $P^{-1}(b)/G[b] \approx M$. (Let A be a collection of sets; then A^* denotes the union of the sets in A.) In this note, we prove that $P^*: Y/G \to B$, defined by $P^*(P^{-1}(b)/G[b]) = b$, is a locally trivial bundle with fiber M. The special cases where M is an i-sphere $(i = 1, 2, 3)$ have been proved in [1] and [2].

The work of Dyer and Hamstrom on completely regular mappings in [1] and [3], that of Kirby and Edwards on local contractibility of spaces of homeomorphism of manifolds in [4] and that of Siebenmann on approximating cellular maps by homeomorphisms in [5] are all essential for the proof of the main result.

2. **Terminology.** All spaces are assumed to be separable metric. A *set K in the interior of an r-manifold is cellular* if $K = \bigcap \{J[n] \mid n = 1, 2, \ldots\}$ where each $J[n]$ is topologically an r-cell and $J[n+1]$ is in the interior of $J[n]$. A *set K in the boundary of an r-manifold is cellular* if it is cellular as a subset of the boundary. A *cellular decomposition* of a manifold is an usc decomposition in which each element is cellular.

REMARK 1. Using this definition of cellular, it is necessary that each decomposition element be entirely in the interior or entirely in the boundary of the manifold; however, this restriction can be avoided by working in the more general setting of *cell-like* decompositions. The proof in this paper holds in this latter setting for $r \neq 3, 4$; but for simplicity, we give only the definition of *cellular* (see [5]).

A map $f: X \to Y$ is *completely regular* provided that, given $\varepsilon > 0$ and $y \in Y$, there is a $\delta > 0$ such that for each point y' within δ of y, there exists a homeomorphism from $f^{-1}(y')$ onto $f^{-1}(y)$ which moves no point as much as ε. A triple (X, f, Y) is a *locally trivial bundle with fiber F* if, for each $y \in Y$, there is an open set U containing y and a homeomorphism $\phi: U \times F \to f^{-1}(U)$ with $f\phi = \pi$ where $\pi: U \times F \to U$ is the projection.

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