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NONNOETHERIAN COMPLETE INTERSECTIONS

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Let all rings here be commutative and unitary. As it is well known, a noetherian local ring A (quotient of a regular ring with residue field K) is a complete intersection if and only if the integers

$$\beta_i = \dim_K \mathrm{Tor}_i^A(K, K)$$

appear in an equality of formal series of the following type

$$\sum \beta_i x^i = (1 + x)^r/(1 - x^2)^s.$$

Furthermore the integer $r - s$ is positive (equal to the dimension of the noetherian ring A). In the nonnoetherian case, by means of André-Quillen homology theory, a criterion is given for characterizing the local rings for which the integers β_i are defined and appear in an equality of formal series as above. An example shows that there is no relation between the integers r and s in the nonnoetherian case.

1. **Result.** Let us consider a local ring A with residue field K and its homological invariants $\mathrm{Tor}_i^A(K, K)$ and $H_j(A, K, K)$ (see [1] for the definition). They are related by the following result, among others.

THEOREM. *All the dimensions β_i of the vector spaces* $\mathrm{Tor}_i^A(K, K)$ *are finite and satisfy an equality*

$$\sum \beta_i x^i = (1 + x)^r/(1 - x^2)^s$$

if and only if all the dimensions δ_j of the vector spaces $H_j(A, K, K)$ are finite and satisfy an equality

$$\sum \delta_j x^j = rx + sx^2.$$

PROOF. The proof is given elsewhere in the paper and involves simplicial theory.

REMARK. In the case of characteristic 0, the theorem is a corollary of a result proved by D. Quillen: The graded vector space $H_*(A, K, K)$ is isomorphic to the graded vector space of the indecomposable elements of the graded Hopf algebra $\mathrm{Tor}_*^A(K, K)$. In the case of characteristic p, such a result cannot hold in all degrees for all rings even if divided powers are considered in the definition of the graded vector space of the indecomposable elements.

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