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SURFACES WITH PARALLEL MEAN CURVATURE VECTOR

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Let M be a surface immersed in a Riemannian manifold R^m of dimension m. Let D denote the covariant differentiation of R^m and $\boldsymbol{n}$ be a normal vector field on M. If we denote by $D^*\boldsymbol{n}$ the normal component of $D\boldsymbol{n}$, then D^* defines a connection in the normal bundle. A normal vector field $\boldsymbol{n}$ is called parallel if $D^*\boldsymbol{n} = 0$.

Let $\boldsymbol{H}$ and $\boldsymbol{h}$ denote the mean curvature vector and the second fundamental form of M in E^m. It is easy to see that minimal surfaces of a euclidean m-space E^m and minimal surfaces of hyperspheres of E^m are surfaces of E^m with parallel mean curvature vector, i.e. $D^*\boldsymbol{H} = 0$. On the other hand, for any analytic function $\varphi \not\equiv 0$ of $z = u + iv$, defined in a neighborhood of the origin in the (u, v)-plane, and constants α, β with $\alpha > 0$, Hoffman [3], [4] proved that, up to euclidean motions and isothermal coordinate $E(u, v)$, locally there exists one and only one surface in E^4, denoted by $M(\varphi, \alpha, \beta)$, with parallel mean curvature vector $\boldsymbol{H}$ such that $\alpha = |\boldsymbol{H}|$, and $\varphi = \varphi_3$, $\beta\varphi = \varphi_4$ where φ_3 and φ_4 are given in the Lemma of [3]. These surfaces are easy to check that they are contained in either an affine 3-space or an ordinary 3-sphere of E^m and they are neither minimal surfaces in E^m nor minimal surfaces of hyperspheres of E^m. Hence, the following problems seem to be interesting.

Problem I. Let M be a surface immersed in a euclidean m-space E^m with parallel mean curvature vector. If M is neither a minimal surface of E^m nor a minimal surface of a hypersphere of E^m, is M contained either in an affine 3-space of E^m or in an ordinary 3-sphere of E^m?

Problem II. If the answer to Problem I is in the affirmative, is M given locally by one of the surfaces $M(\varphi, \alpha, \beta)$?

The main purpose of this paper is to announce the following results. The details will appear elsewhere.

THEOREM I. *The answer to Problem* I *is in the affirmative.*

THEOREM II. *The answer to Problem* II *is in the affirmative.*

From theorem I we have the following corollaries.

COROLLARY 1. *Let M be a surface immersed in an m-sphere S^m with*

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